Government Support for SMEs in Response to COVID-19: Theoretical Model Using Wang Transform

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Background

- Relative to established corporations, Enterprise Risk Management (ERM) for SMEs still have huge growth potential
- Integral role of SMEs in economy
- SMEs face higher difficulty in obtaining bank loans
- Problem exacerbated due to COVID-19
- Note: This is a fairly theoretical paper but I'll try to focus more on the key ideas instead
- Also, interestingly, after our paper has been written, we observed that several measures implemented by governments globally are similar to the ones proposed in this paper.

Overview

1. Brief background on Wang Transform
2. Theoretical Approach
   - Double Process for SME, Structural Model (Loan Default vs Intrinsic Value)
3. Application of theoretical models to evaluating efficacy of practical proposed measures:
   - Bridge loans (e.g., Temporary Bridging Loan Programme), insurance for bank loans (e.g. loan guarantees, Japan Federation of Credit Guarantee), interest rate subsidy (e.g. "MAS SGD Facility for ESG Loans"), relief of tax burdens (e.g. tax cuts/deferments)
4. Construction of confidence intervals for drift term
5. Application to Bank's underwriting capability
6. Empirical evidence

Wang Transform

- Let X be a random loss due to default of a bank loan, which can be described by a loss curve given by $S_X(w) = P\{X > wL\}$, where w is percentage of the outstanding loan amount L.
- Applying the Wang Transform to convert loss curve $S_X(w)$ to price curve $S_X^*(w)$:

$$S_X^*(w) = \Phi\left[\Phi^{-1}\left(S_X(w)\right) - \lambda\right]$$

- where λ is defined as market price of risk which captures the underlying systematic risk, it is fairly obvious the Φ is the Gaussian CDF.

Wang Transform

- Risk Margin is simply: $E^*[X] - E[X] = L \cdot \int_0^1 [S_X^*(w) - S_X(w)]dw$
- Note: Wang Transform shown does the conversion without reflecting an increasing sampling errors at tails of distributions
- To incorporate these sampling errors, we may apply the two-factor Wang Transform:

$$S^*(x) = Q\left[\Phi^{-1}(S(x)) + \lambda\right]$$

- where Q follows a Student-t distribution with k degrees of freedom
- Empirically, Wang Transform has been shown to perform well in explaining real-world risk pricing data (Wang (2000, 2002, 2003, 2004); Kijima & Muromachi (2008); Glasserman (2007))

Theoretical Approach

- We define a Firm's Book Value = Asset – Liability
- Book Value $\neq$ Intrinsic Value
- IV is proposed as a combination of the firm's BV and expectation for future profitability, where the latter is defined as Franchise Value (FV)
- Due to COVID-19, cashflows issues arise & a firm's BV is diminished, but expectation of potential development in the future (FV) should not be diminished, given as:

$$FV = \sum_{j=0}^{\infty} \frac{\left(1-\tau(\theta_i)\right) \cdot Income[t+j, t+j+1]}{\left(1+r(\theta_i)\right)^j}$$

Theoretical Approach

- Double Process: An SME is subjected to
  1. IV which follows a Geometric Brownian Motion, with parameters (μ, σ)
  2. BV liquidity crunch due to events such as COVID-19
- Implication
  - A firm may suffer from a liquidity crunch despite having a positive IV (which subsequently disappears when the firm shuts down the business due to the liquidity crunch)

Theoretical Approach

- Consider a single period model $[t = 0, t = T]$, the terminal intrinsic value would naturally be lognormally distributed
- We employ the distance to default model to connect the drift parameter and the default probability, as follows:

$$ln(IV_{terminal}) \sim N(\mu T + ln(IV_0), \sigma^2 T)$$

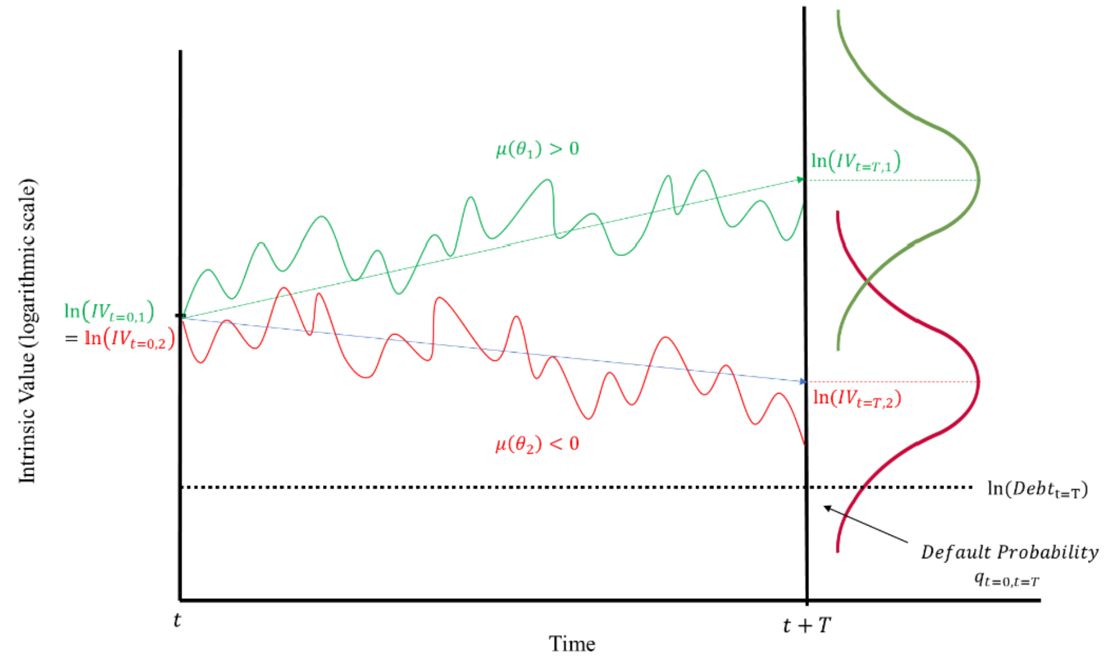

Figure 3. Illustration of the distance-to-default model for one firm with different drift terms

Theoretical Approach

- **Using Merton Model, default probability from projected IV is given as**

$$q_{0,T} = \Phi\left(\frac{ln(Debt) - ln(IV_{terminal})}{\sigma\sqrt{T}}\right)$$

- **Our structural equation is thus**

$$\Phi^{-1}(q_{0,T}) = \frac{ln(Debt) - \mu T - \frac{\sigma^2 T}{2} - ln(IV_0)}{\sigma\sqrt{T}}$$

Theoretical Approach

- Finally, we show that the objective default probability q and the risk-neutral probability q^* is connected by the Wang Transform formula as follows

$$q^*_{t=0,t=T}(\theta_i) = \Phi\left[\Phi^{-1}\left(q_{t=0,t=T}(\theta_i)\right) - \lambda_T\right]$$

- where $\lambda_T = \frac{\mu - r}{\sigma}\sqrt{T}$

Application – Bridge Loans

- We define the liquidity crunch when BV falls below 0.
- The estimated probability of a liquidity crunch for the i^{th} SME is given as follows:

$$q(\theta_i) = 1 - F_T\big(C_0(\theta_i)\big) = exp\left(-\left(\frac{C_0(\theta_i)}{\beta}\right)^k\right)$$

- Net Value Creation brought about by Bridge Loans is defined as follows:

$$Net\ Value\ Creation = Value\ Creation\ - Opp\ Cost$$

- Where

$$Value\ Creation = \left\{\exp\left(-\left(\frac{C_0(\theta_i)}{\beta}\right)^k\right) - \exp\left(-\left(\frac{C_0(\theta_i)+E}{\beta}\right)^k\right)\right\} \cdot Comm\ (\theta_i)\Big|_{t=1}$$

$$Opp\ Cost = \omega \cdot E \cdot \frac{Expenditure[t, t+1]}{12}$$

Application – Bridge Loans

- Applying the Weibull to model the "containment" measures, our model makes the case for emergency bridge loans to be made to SMEs.
- More interestingly, our model demonstrates that the optimal level of bridge loans
- Exceedance threshold exists
- Supply side of economy can recover fairly soon, but demand side may take longer to recover. Our model can capture this uncertainty via the shape k.

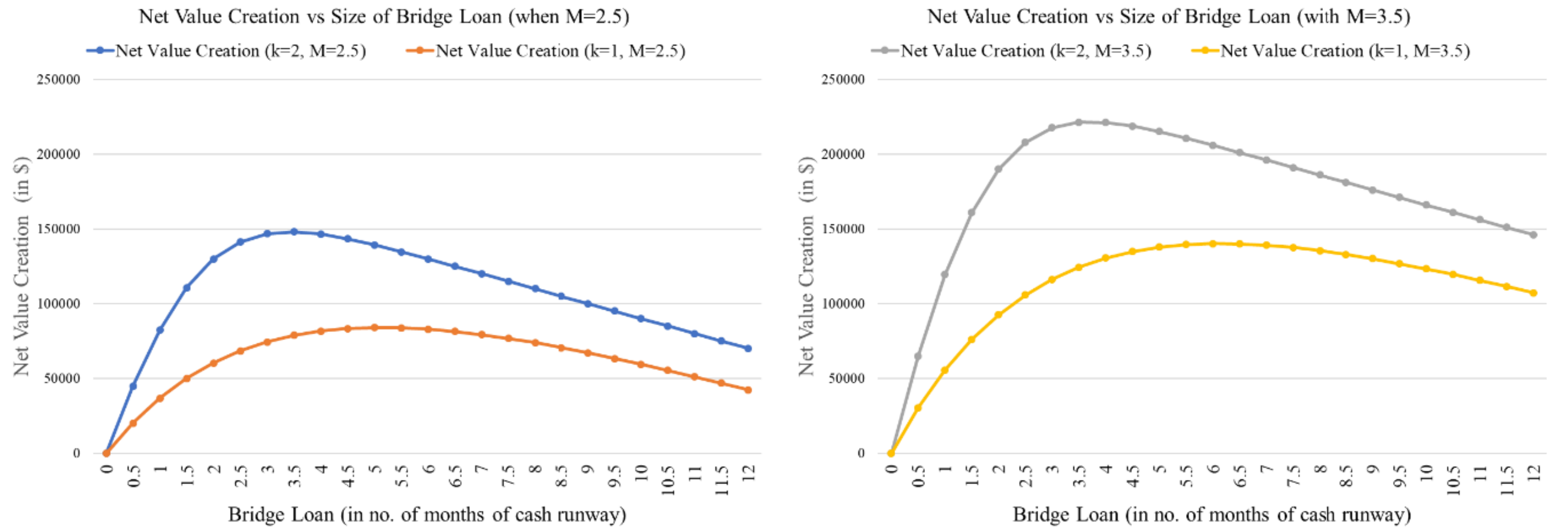

Figure 4. Net Value Creation vs Bridge Loan (with different shape parameters), and with $M = 2.5$ (Left-Panel) and $M = 3.5$ (Right-Panel).

Application – Insurance Pool for Bank Loans

- Government funded pool that provides insurance for loans made to SMEs by participating banks
- Proportional insurance is a common implementation in the real-world

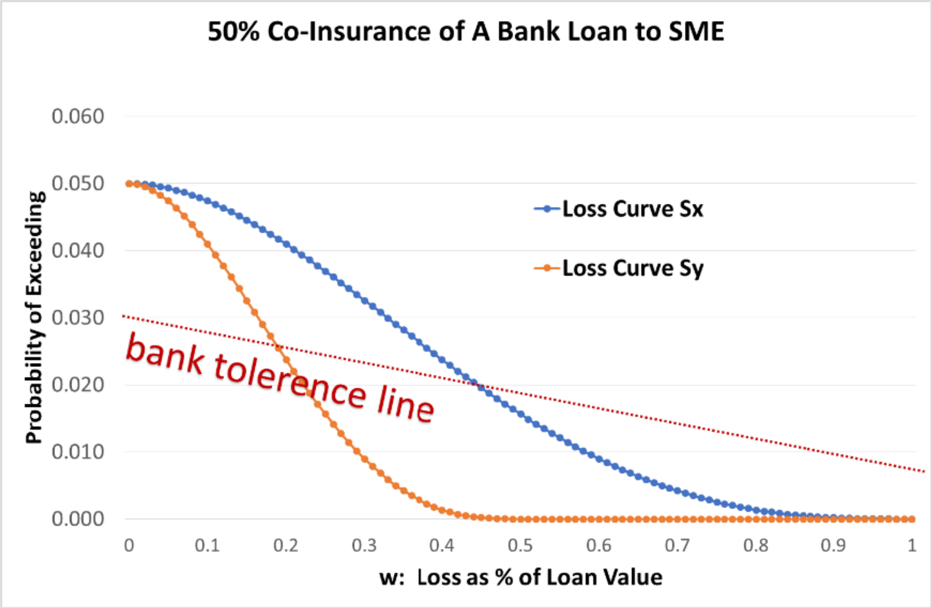

Figure 5: Illustration of the Probability of Exceedance against the Loss as % of Loan Value, under a 50% Co-Insurance of a Bank Loan to SME.

Application – Insurance Pool for Bank Loans

- Our model suggests that proportional insurance of bank loans provides some but generally weak incentives for banks to lend to SMEs (i.e. not the most efficient methodology).
- Proportional insurance is triggered only after bank loan default, i.e. it does not reduce the default probability
- Local branch bank managers generally are evaluated and penalized based on the NPL
- Certain conditions for banks to participate in insurance pool is the overall loan portfolio default ratio does not exceed a given threshold (e.g. 3%).
- Instead, a Government subsidy that pays banks an adequate level of i/r is preferred.
- Question is: What is the benchmark price of default risk for bank loans to SMEs?

Application - I/R Subsidy for Bank Loans

- Purpose: Identify appropriate level of interest rate subsidy via Wang Transform
- Applying two-factor Wang Transform converting real-world estimated loan default probability q, to risk-neutral loan default probability q^*

$$q^* = Q\left[\Phi^{-1}(q) - \lambda\right]$$

- We show that the interest rate margin is thus

$$\int_0^1 [S_Y^*(y) - S_Y(y)]\, dy = (q^* - q)$$

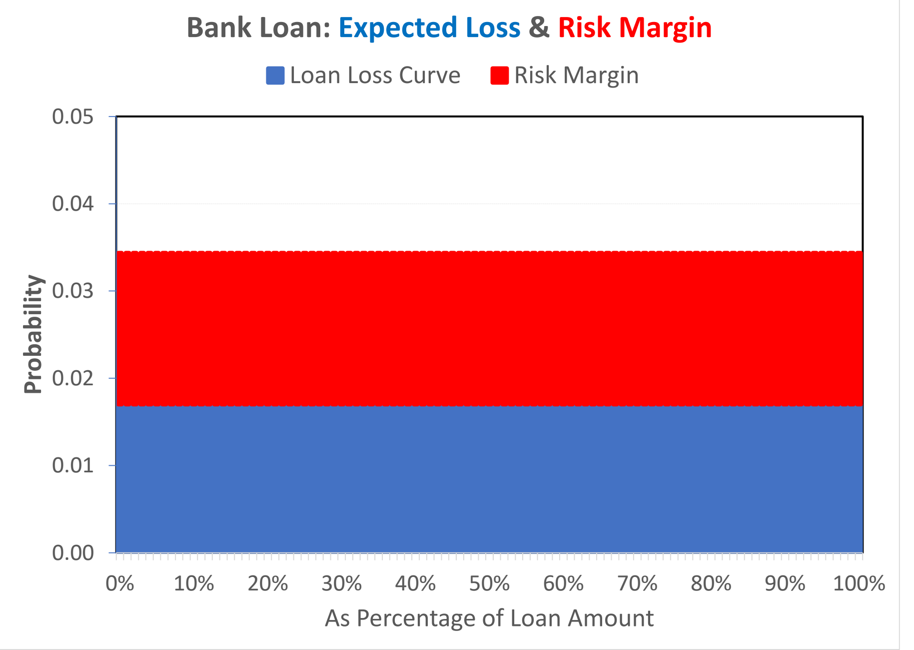

Figure 6: An illustration of Wang Transform from Loss Curve S to Price Curve S^*, where the plot is Loss Exceedance Probability against the Loss as % of Loan Value, assuming that in the event of default, the recovery rate is zero.

Application - I/R Subsidy for Bank Loans

- Using Wang Transform to calculate the appropriate level of interest rate subsidy
- Our model provides a tractable method for governments to determine an appropriate i/r subsidy to optimally incentivize banks to issue more bank loans to SMEs.
- Generally speaking:
  - Government pays for no less than the risk margin
  - SME to pay for no more than the expected loss
  - Both are calculated under a risk-neutral setting

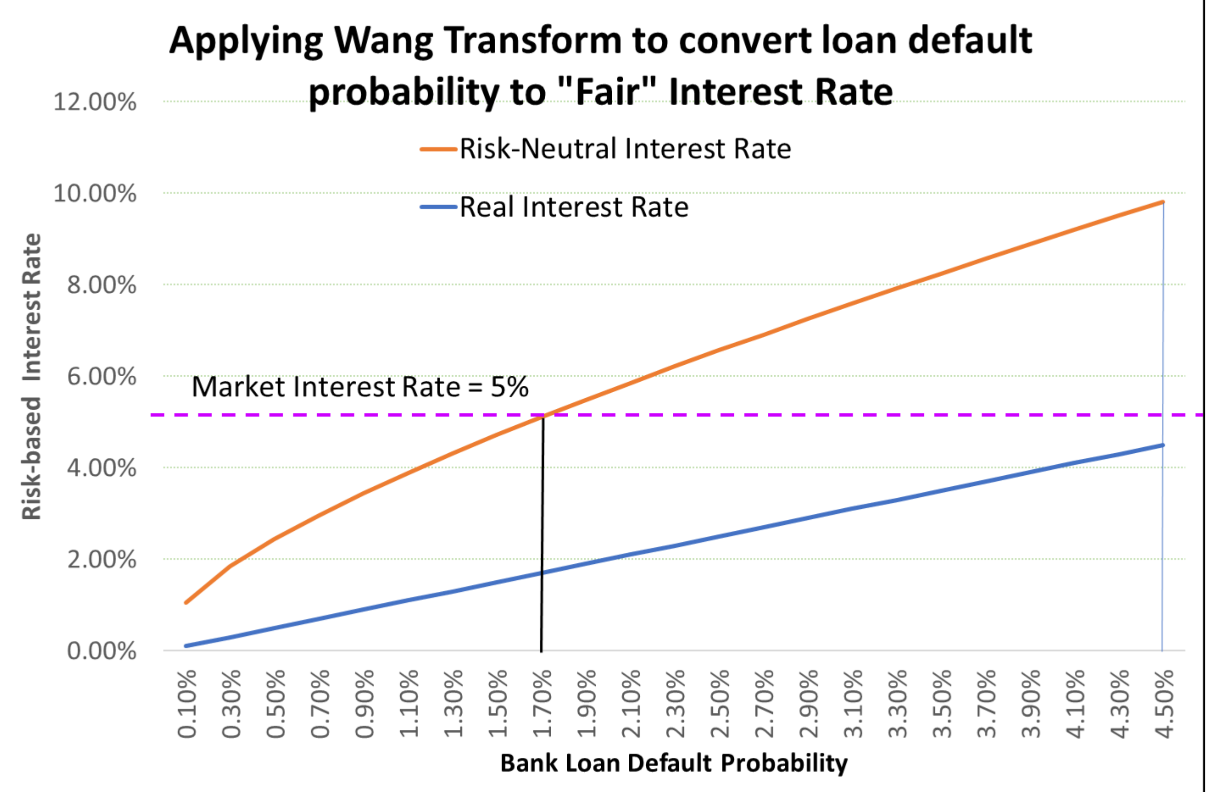

Figure 7: Illustration of the loan default probability and the associated real interest rate and risk-neutral interest rate, calculated using the Wang Transform.

Application – I/R Subsidy for Bank Loans

- In China, a type of govt support includes i/r subsidy paid to SMEs.
- We recommend the i/r subsidy be paid to banks instead.
- This will have a leverage effect on increasing number of bank loans to SMEs.

- In Singapore, a type of govt support is the MAS SGD Facility, which lowers the cost of borrowing by banks (i.e. similar to an i/r subsidy to banks).
- The lowered cost of borrowing is level.
- Our model allows for individualized calculation of i/r subsidy for different SMEs, i.e. maximizing efficiency of government subsidies.

Application - Tax Relief

- Defining the community value of an SME as follows:

$$CommValue\,(\theta_i)\Big|_t = \left[\frac{\tau(\theta_i)}{1-\tau(\theta_i)}\right] FranchiseValue(\theta_i)\Big|_t$$

- We show that an optimal effective tax rate exists.
- Intuition:
  - Too high → disincentive to companies which lead to a reduction in CV
  - Too low → govt have insufficient tax revenue to maintain basic infrastructure
- Wang Transform provides a tractable method to calculate the optimal tax rate and tax relief for SMEs

Confidence intervals for drift parameter

- Our models make no assumptions on efficient market, but rather acknowledge that there are limited info and inherent uncertainty in estimating the drift term for a firm.
- Using the structural models developed earlier, we provide a tractable expression for estimating the confidence interval for the drift term, as follows:

$$\boldsymbol{\Phi}^{-1}\left(q_{t=0,t=T}^{lower}(\theta_i)\right) = \frac{ln(Debt(\theta_i)|_{t=T}) - \mu^{upper}(\theta_i)T - \sigma^2(\theta_i)T/2 - ln\big(IV_{t=0}(\theta_i)\big)}{\sigma(\theta_i)\sqrt{T}}$$

$$\boldsymbol{\Phi}^{-1}\left(q_{t=0,t=T}^{upper}(\theta_i)\right) = \frac{ln(Debt(\theta_i)|_{t=T}) - \mu^{lower}(\theta_i)T - \sigma^2(\theta_i)T/2 - ln\big(IV_{t=0}(\theta_i)\big)}{\sigma(\theta_i)\sqrt{T}}$$

Confidence intervals for drift parameter

- Our models provide banks a method to estimate the CI of the drift term of a firm.
- Given that the drift term is a good proxy for the underlying future potential prospect of the firm, this provides a novel methodology for banks to underwrite risk of SMEs.
- This is interesting because it is important for banks to have a tractable methodology to differentiate between the “good” and “bad” firms

Application – Bank's Underwriting Capability

- We define that a bank's underwriting capability is captured by the variation parameter δ, applied in the Wang Transform formula, where naturally

$$\delta_{strong\ underwriting} < \delta_{weak\ underwriting}$$

- Practically speaking, we note that δ may be bounded by a floor, and this arises from the inherent unknown-unknowns (or simply due to limitation of available info).
- We run a simulation study, where we assume 1000 good and 1000 bad quality firms, where the good firm has a lower default probability, and a strong bank which has a lower δ than a weak bank.

Application - Bank's Underwriting Capability

- **Simulation Results:**
- **We observe that based on our model, a bank with strong underwriting capability has a lower observed default rate**
- **We observe the driving force behind this is largely attributable to the strong bank's ability to identify the bad firms much better than the weak bank**
- **Difference in ability between strong and weak bank to identify good firms are not as significant.**

	Sample Size	# of Loans Made	# of Loans Not Made	Expected # of Default	Observed Default Rate
Good Firm ($q(\theta_{Good}) = 0.02$)	1000	630 (TP)	370 (FN)	12.6	0.02
Bad Firm ($q(\theta_{Bad}) = 0.10$)	1000	113 (FP)	887 (TN)	11.3	0.10
Total	2000	**743**	**1257**	**23.9**	**0.0322**

Table 3. Bank with weak underwriting capability, where $\delta_{weak\ bank}(\theta_i) = 0.5$

	Sample Size	# of Loans Made	# of Loans Not Made	Expected # of Default	Observed Default Rate
Good Firm ($q(\theta_{Good}) = 0.02$)	1000	746 (TP)	254 (FN)	14.92	0.02
Bad Firm ($q(\theta_{Bad}) = 0.10$)	1000	8 (FP)	992 (TN)	0.80	0.10
Total	2000	**754**	**1246**	**15.72**	**0.0208**

Table 4. Bank with strong underwriting capability, where $\delta_{strong\ bank} = 0.25$

Application – Bank's Underwriting Capability

- <u>Overlap Index</u>
- We plot the estimated default pdf.
- We define the differentiation index

$$DI = \frac{1}{2}(Red + Blue)$$

- Overlap index is defined as cumulative probability of good firms and bad firms with the same estimated default probability, given as

$$OI = 1 - DI$$

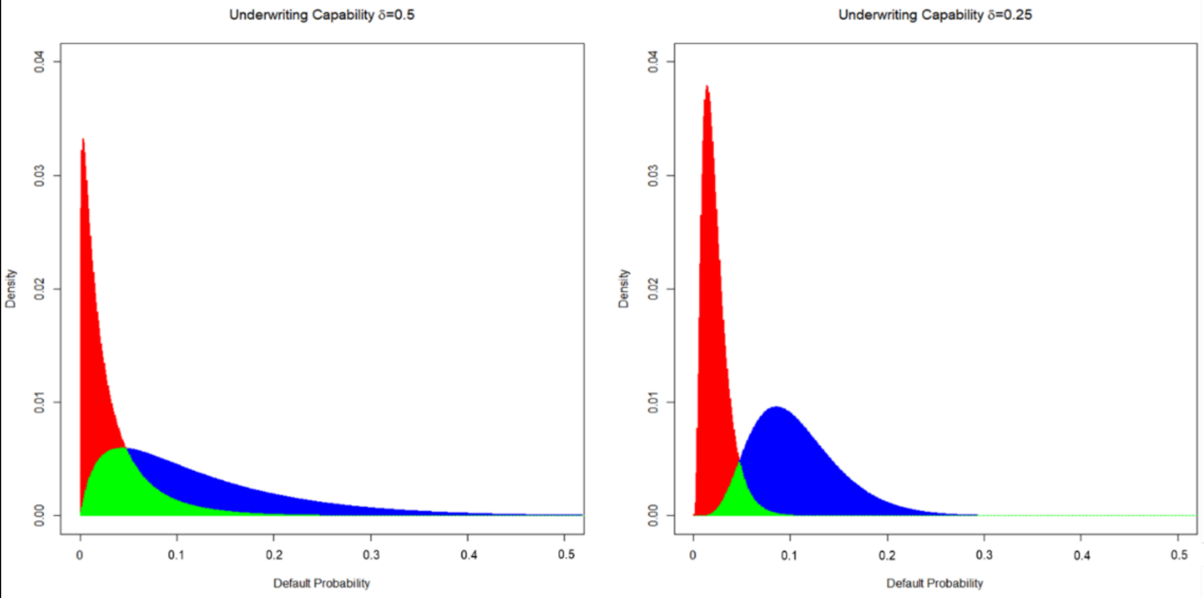

Figure 9. Overlapping Regions in Loans to Good Firms and Bad Firms by banks with different underwriting capability. Left panel: bank with weak underwriting capability; right panel: bank with strong underwriting capability.

- OI is a novel proxy proposed by our paper that captures the difficulty the bank has in differentiating between good firms and bad firms.

Application – Bank's Underwriting Capability

- Per our simulation example, in line with our expectation, OI for:
  - Strong bank → 12.25%
  - Weak bank → 44.00%
- OI vs conventional measure (such as NPL)
  - OI is an ex-ante measure (i.e. before any classification is done)
  - Conventional measures tend to be ex-post in nature

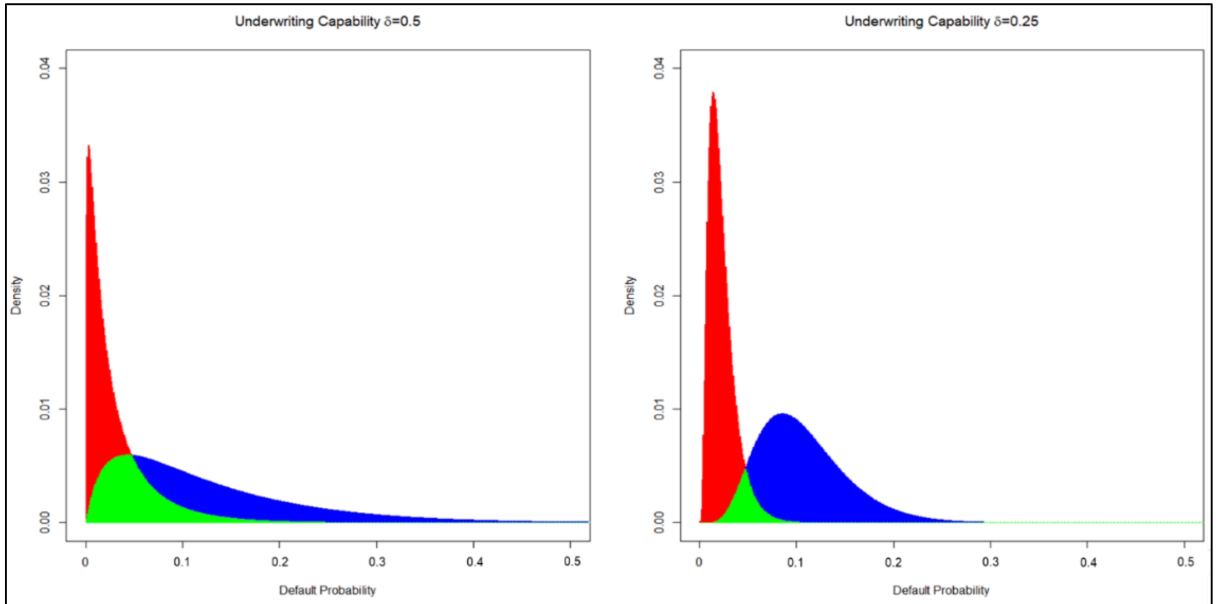

Figure 9. Overlapping Regions in Loans to Good Firms and Bad Firms by banks with different underwriting capability. Left panel: bank with weak underwriting capability; right panel: bank with strong underwriting capability.

Empirical Evidence

- Data: Default rate figures of Chinese Banks from the CBIRC
- Applying Wang Transform to calculate the risk-neutral i/r, we observe interestingly:
  - Risk-neutral i/r for both types of commercial banks < prevailing i/r of 5%
  - Risk-neutral i/r for both types of community banks > prevailing i/r of 5%

Type of Banks	Default Rate of all bank loans	Real Interest Rate	Risk-Neutral Interest Rate
Large Commercial Banks	1.39%	1.39%	4.49%
Commercial Banks	1.64%	1.64%	4.99%
City Community Banks	2.49%	2.49%	6.55%
Rural Community Banks	4.09%	4.09%	9.18%

Table 7. Reported Bank Loan Default Rates for the 1st Quarter 2020 and the risk-neutral interest rate calculated using Wang Transform

Empirical Evidence

- **Implications**
  - **Commercial banks generally have access to larger or more established companies**
  - **More i/r subsidy should be allocated to community banks**
  - **Potentially because community banks have a higher concentration of SMEs who face greater difficulty in accessing bank loans.**

Conclusion

- Applying Wang Transform in a theoretical framework
- Evaluating the efficacy of various government measures in light of COVID-19 pandemic
- Novel measures proposed to support the bank's underwriting capabilities
- Enterprise Risk Management (ERM) for SMEs still have huge growth potential which may play an important role in improving bank's underwriting result of SMEs.

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