Wrong Data & Right Model, Right Data & Wrong Model, No Data? – Examples from Reinsurance

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Situation 1 - No Data?

- No loss observations (rare catastrophic events)
- Changing exposures
- Changing probabilities (climate change, sea levels,...)

  - **Focus Exceedance Probability Curves**

Exceedance Probability - Definitions

Occurrence Exceedance Probability (OEP) The OEP is the probability that at least one loss exceeds the specified loss amount.

Aggregate Exceedance Probability (AEP) The AEP is the probability that the sum of all losses during a given period exceeds some amount.

Conditional Exceedance Probability (CEP) The CEP is the probability that the amount on a single event exceeds a specified loss amount; this is equal to 1-CDF of the severity curve as used by actuaries in other contexts.

Reference: A New Approach to Managing Risks – Grossi, P and Kunreuther, H

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Focus of next example

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OEP – Example XL Pricing

Given: turquoise columns

Average Return Period (Years)	**Probability of Non-Exceedance** (Cdf)	**Gross Loss OEP** (Monetary Amount)
2,000	99.95%	404,108
1,000	99.90%	215,147
500	99.80%	114,518
400	99.75%	93,471
250	99.60%	60,928
200	99.50%	49,718
100	99.00%	26,416
80	98.75%	21,541
50	98.00%	14,002
20	95.00%	6,003
10	90.00%	3,120

OEP – Example XL Pricing

Given: turquoise columns
Terms: for each claim the XL pays the excess of 20k with limit 100k per claim

Average Return Period (Years)	**Probability of Non-Exceedance** (Cdf)	**Gross Loss OEP** (Monetary Amount)	**Loss to XL 100'000 xs 20'000**	**Probability - Actuary A**
2,000	99.95%	404,108	100,000	0.05%
1,000	99.90%	215,147	100,000	0.10%
500	99.80%	114,518	94,518	0.05%
400	99.75%	93,471	73,471	0.15%
250	99.60%	60,928	40,928	0.10%
200	99.50%	49,718	29,718	0.50%
100	99.00%	26,416	6,416	0.25%
80	98.75%	21,541	1,541	0.75%
50	98.00%	14,002	-	3.00%
20	95.00%	6,003	-	5.00%
10	90.00%	3,120	-	90.00%
			Expected Loss to XL	**525**

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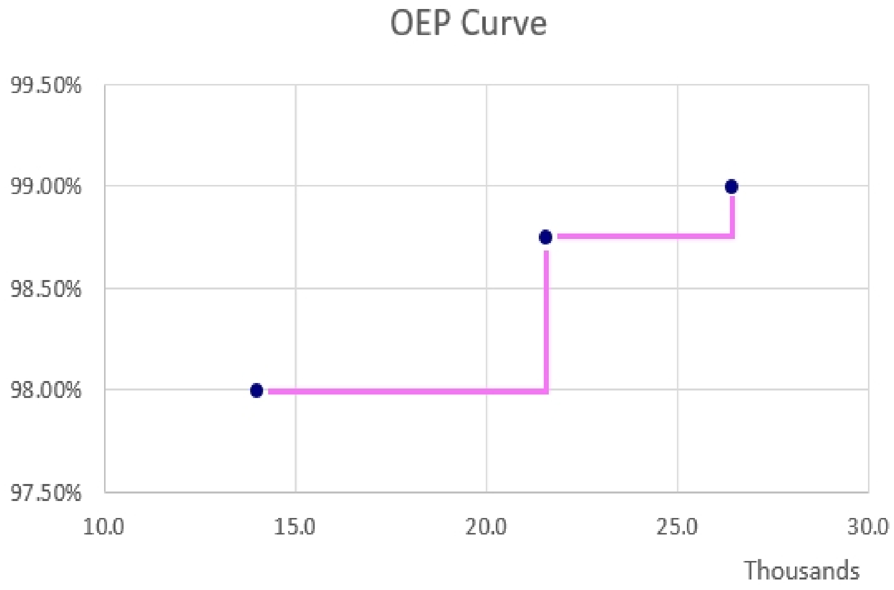
10	90.00%	3,120	-	90.00%	5.00%
			Expected Loss to XL	**525**	**358**

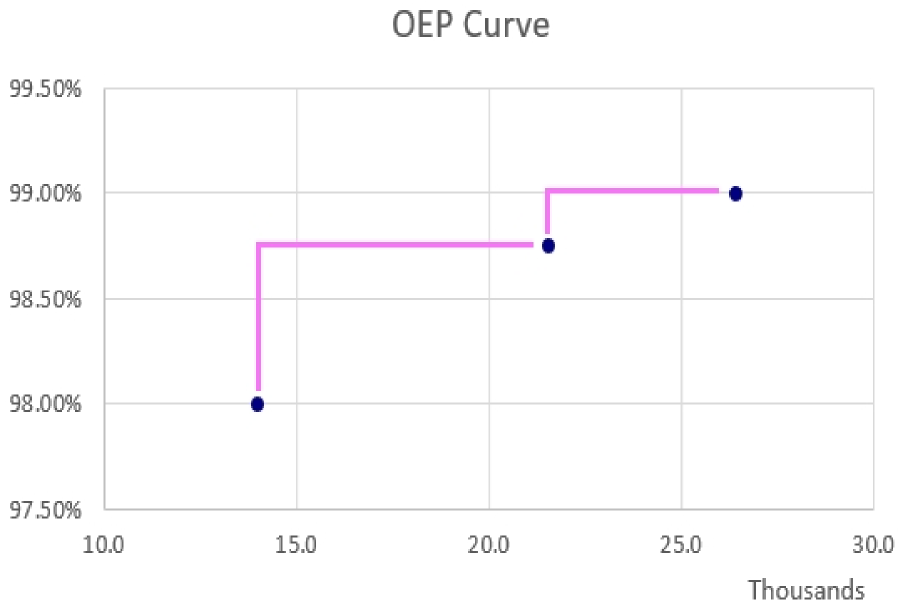
OEP – Example XL Pricing

Different interpolations may lead to very different results

Actuary A did...



...whereas actuary B did



Situation 2 – We got data. Is the model right?

- Still focus OEP

If actuaries A and B were given the entire OEP curve, their differences would go to zero.

Will their expected losses converge to the ‘true’ expected loss of the XL?

OEP – ‘Formulating’ the Definition

We remind the definition

“The OEP is the probability that at least one loss exceeds the specified loss amount”

Hence

OEP(loss amount) = 1 - Probability(no loss exceeds the loss amount)

= 1 - Probability (largest loss does not exceed the amount)

OEP – ‘Formulating’ the Definition – iid Losses

Random number N of events

$$X_1, X_2, \ldots, X_N > 0$$

iid with cdf $F(x) > 0$ for $x > 0$

$$\mathbb{P}^{OEP}(x) \;=\; \mathbb{P}\left(\max(X_1, X_2, \ldots, X_N) \leq x\right)$$

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$$\begin{aligned}\mathbb{P}^{OEP}(x) &= \mathbb{P}\left(\max(X_1, X_2, \ldots, X_N) \leq x\right) \\ &= \sum_{k=0}^{\infty} (F(x))^k \mathbb{P}(N = k) \\ &= \begin{cases} \mathbf{1}_{\{x=0\}}\mathbb{P}(N=0) + \mathbf{1}_{\{x>0\}} M_N(\log(F(x))) & \text{if } F(0) = 0 \\ M_N(\log(F(x))) & \text{if } F(0) > 0 \end{cases}\end{aligned}$$

where

$$M_N(u) = \mathbb{E}[e^{uN}]$$

and $\mathbf{1}_{\{\}}$ is the indicator function.

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The expectation with the entire OEP curve is

$$\mathbb{E}[OEP(X)] = \int_{\{x>0\}} x\, d\mathbb{P}^{OEP}(x)$$

$$= \int_{\{x>0\}} x\, \mathbb{E}[N e^{\log(F(x))(N-1)}] dF(x).$$

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Since for $N \geq 1$

$$\log(F(0))(N-1) \leq \log(F(x))(N-1) \leq 0$$

we find

$$M_N\left(\log(F(0))\right) \mathbb{E}[N] \int_{\{x>0\}} x \, dF(x) \leq \ldots \tag{0.1}$$

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$$\begin{aligned} M_N\left(\log(F(0))\right)\mathbb{E}[N] \int_{\{x>0\}} x \, dF(x) &\leq \dots \\ \dots \mathbb{E}[OEP(X)] &\leq \mathbb{E}[N] \int_{\{x>0\}} x \, dF(x). \end{aligned} \tag{0.1}$$

The upper bound is the true expected loss.

If N is Poisson(λ) then (0.1) becomes

$$\exp\left(\lambda(F(0)-1)\right) * \text{true expected loss } \leq \ldots$$
$$\ldots \mathbb{E}[OEP(X)] \leq \text{ true expected loss.}$$

OEP Example – Let's ask the 'true' model!

1) Number of claims above 3'000, Poisson with mean 0.11

2) Claims distributed Pareto with cdf $\mathrm{F}(x) = 1 - \left(\frac{3000}{x}\right)^{1.1}, x \geq 3000$.

➢ *used to create the OEP example*

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The expected loss to a 100'000 xs 20'000 XL is = **447.8**

For the error estimate (0.1) we obtain

exp(-0.11(3000/20'000)$^{1.1}$) *447.8 = **441.7** ≤ ...

... OEP Expectation ≤ **447.8**

OEP Example – Let’s ask the ‘true’ model!

Back to our actuaries A and B

Averaging their expected losses

gives (525+358)/2 = **441.5**

Dividing this by 98.75% (from OEP table) gives = **447.1**

OEP Example – Let’s ask the ‘true’ model!

Back to our actuaries A and B

Averaging their expected losses

gives (525+358)/2 = **441.5**

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Alternative way:

Estimate the underlying model using the OEP data

Situation 3 - Wrong Data?

Suppose we have enough historical data and want to price a treaty covering large losses.

The large losses are described by...

1) a frequency (how many losses?)
2) a severity (how large are the losses?)

- **Focus Frequency**

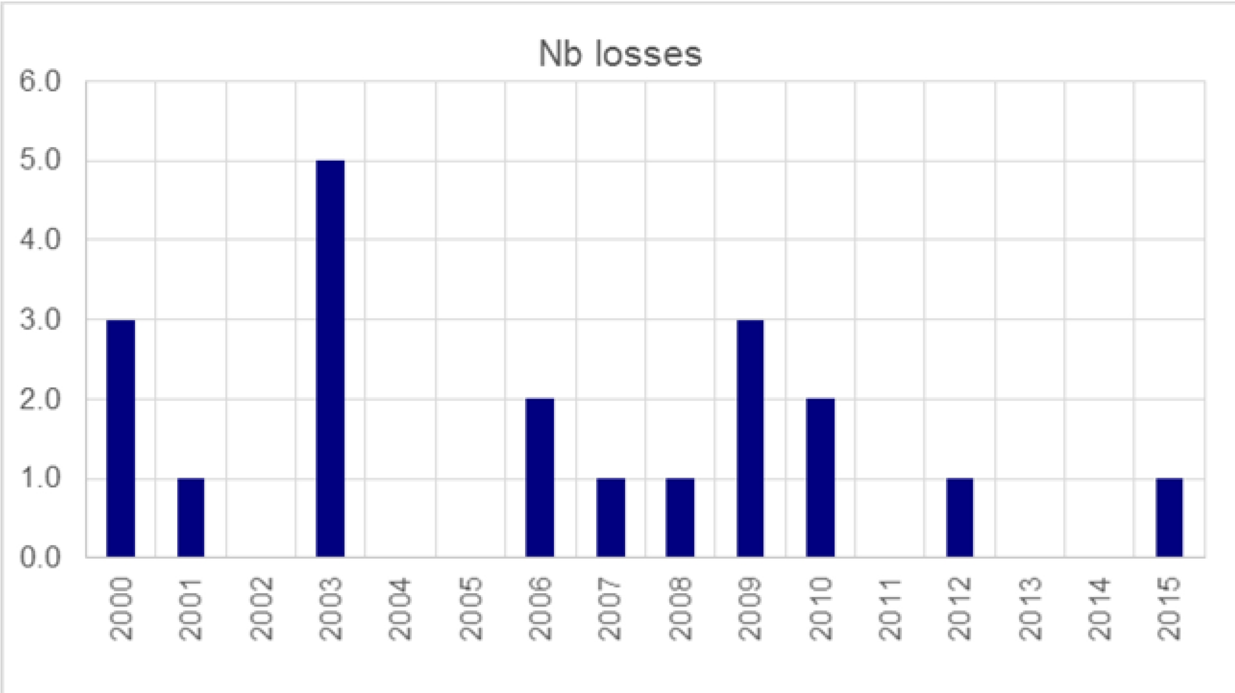
Example Frequency

Data – large losses

2000-2015

Year	Incurred
2000	632,812
2000	2,023,961
2000	698,979
2001	638,449
2003	1,478,491
2003	1,782,429
2003	544,000
2003	3,068,898
2003	504,120
2006	545,210
2006	1,309,699
2007	1,155,370

Number of large losses



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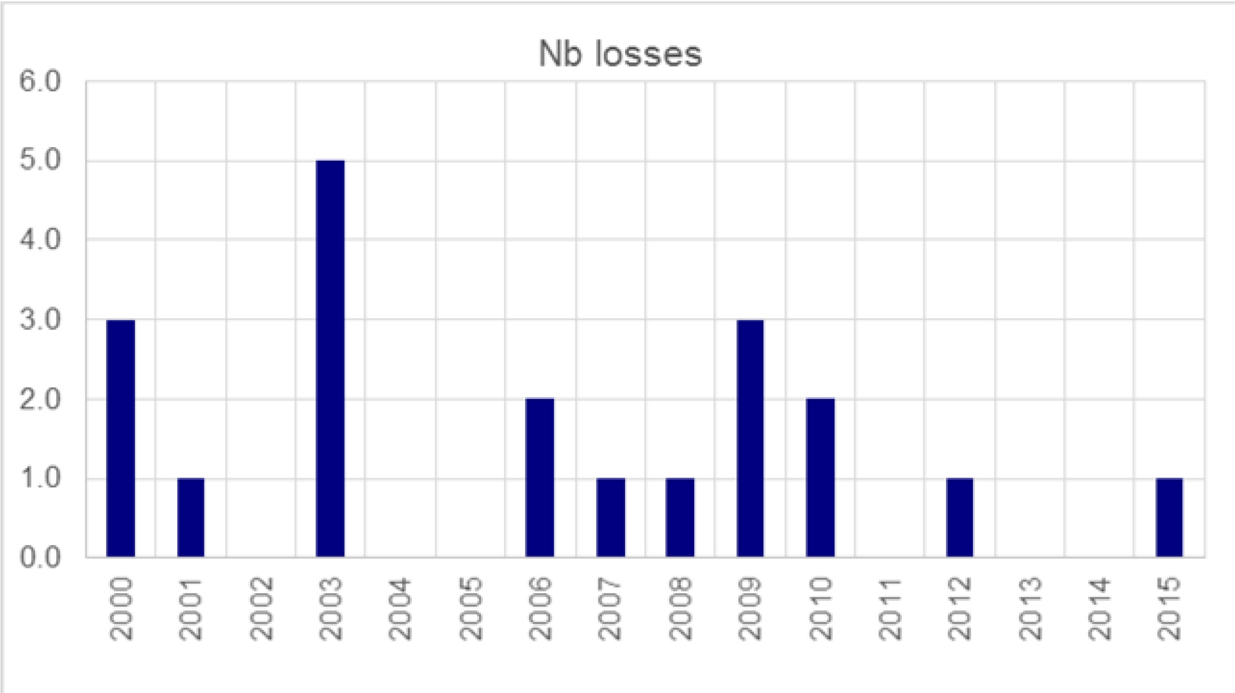

	2000-2015	2000-2007	2008-2015
Avg nb losses	1.3	1.5	1.0

Example Frequency

- Shift in portfolio?
- Legal framework?
- Underwriting guidelines?
- Policy holder’s deductibles?
- Vulnerability?

Example Frequency

Treaty to price

Aggregate Limit	Limit	Priority	Deductible
5.0m	2.0m	1.0m	2.0m

Example Frequency

Treaty to price

Aggregate Limit	Limit	Priority	Deductible
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Actuary A

Frequency with mean = 1.3

Claims ~ Pareto

Obtains:

Example Frequency

Treaty to price

Aggregate Limit	Limit	Priority	Deductible
5.0m	2.0m	1.0m	2.0m

Actuary A

Frequency with mean = 1.3

Claims ~ Pareto

Obtains:

EL = 28.1k

Stdev = 207k

Price = 48.8

Example Frequency

Treaty to price

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5.0m	2.0m	1.0m	2.0m

Actuary A

Frequency with mean = 1.3

Claims ~ Pareto

Obtains:

EL = 28.1k

Stdev = 207k

Price = 48.8

Actuary B

Frequency with mean = 1.3

Claims ~ Pareto (same as actuary A)

Obtains:

Example Frequency

Treaty to price

Aggregate Limit	Limit	Priority	Deductible
5.0m	2.0m	1.0m	2.0m

Actuary A

Frequency with mean = 1.3

Claims ~ Pareto

Obtains:

EL = 28.1k

Stdev = 207k

Price = 48.8

Actuary B

Frequency with mean = 1.3

Claims ~ Pareto (same as actuary A)

Obtains:

EL = 20.1k

Stdev = 164k

Price = 36.5

Example Frequency

Treaty to price

Aggregate Limit	Limit	Priority	Deductible
5.0m	2.0m	1.0m	2.0m

Actuary A

Frequency w

Claims ~ Par

Actuary A's EL is 40% higher!

Actuary A is 34% more expensive!

Obtains:

EL = 28.1k

Stdev = 207k

Price = 48.8

Stdev loading 10%

Obtains:

EL = 20.1k

Stdev = 164k

Price = 36.5

Example Frequency

Here is why

Both actuaries did not make a trend correction, BUT

- Actuary B used a Poisson with mean 1.3
- Actuary A used a Negative Binomial with mean 1.3 and variance 2.0
- Leverage effect of deductible

Questions ?

Thank you !

With You, For You

Daniel is the director of ProMaSta Pte Ltd, has a MSc in Mathematical Statistics, a PhD in Economics and is a fully qualified actuary of the Swiss Association of Actuaries.

Prior to founding ProMaSta, Daniel worked in a reinsurance company and was responsible for the evaluation of reinsurance contracts and insurance-linked securities (catbonds) as well as for the development, maintenance and improvement of the pricing tools.

Besides, Daniel has published in several peer reviewed research journals and acts as a reviewer.

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