Premium Liability Correlations and Systemic Risk

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Introduction

- Based on research I am doing for my Masters thesis

- How to measure correlations between premium liability and outstanding claims
- How to measure the split between systemic risk versus non-systemic risk

The Model

For each and every claim:

$$S = A + B \times e_1 + C \times e_2$$

actual claim size

average claim size

systemic risk – random effect shared by cohorts of claims

non-systemic / claim specific risk

Runoff Triangles

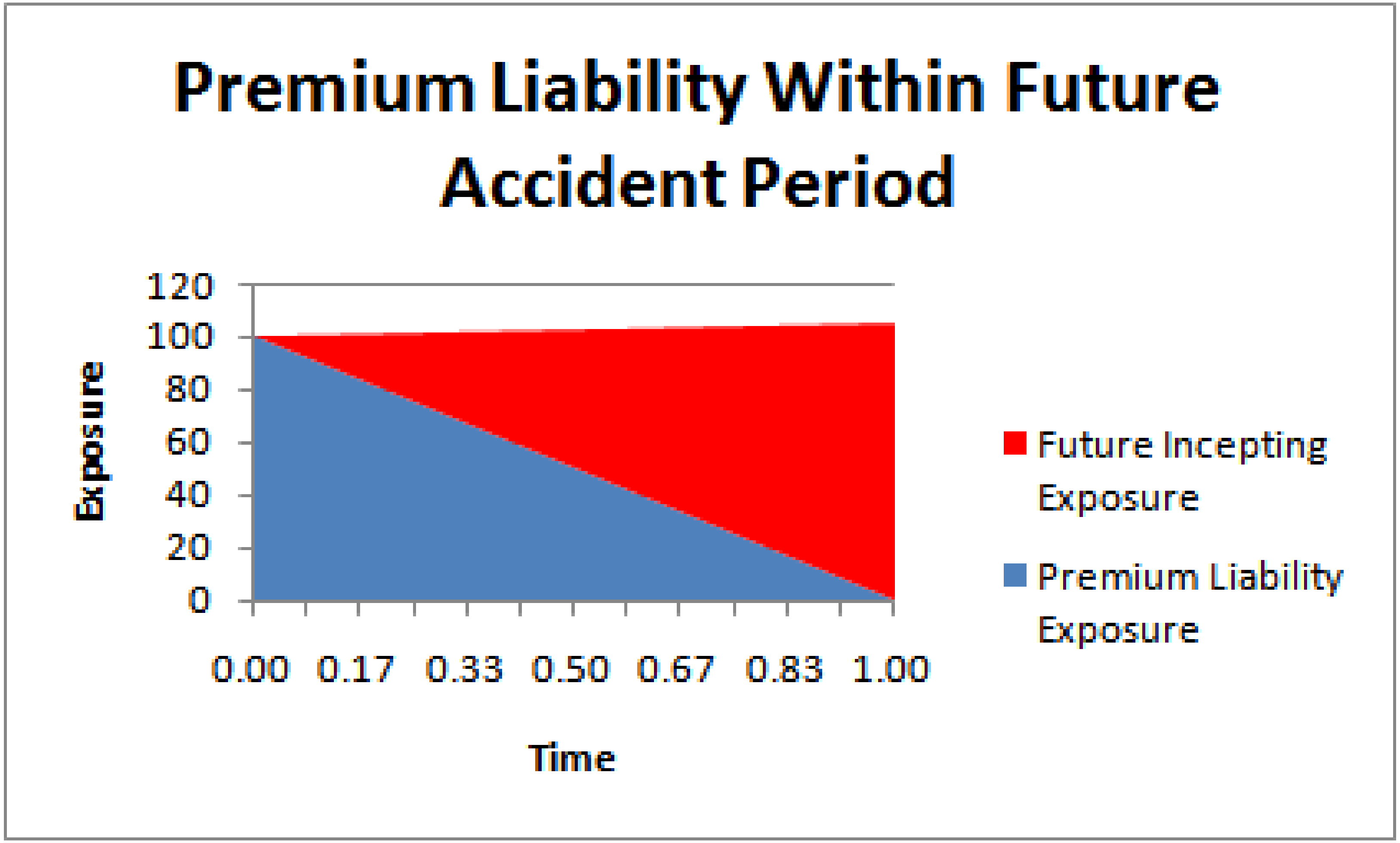
P	P	P	P	P	P	P	P	P	O
P	P	P	P	P	P	P	P	O	O
P	P	P	P	P	P	P	O	O	O
P	P	P	P	P	P	O	O	O	O
P	P	P	P	P	O	O	O	O	O
P	P	P	P	O	O	O	O	O	O
P	P	P	O	O	O	O	O	O	O
P	P	O	O	O	O	O	O	O	O
P	O	O	O	O	O	O	O	O	O
F	F	F	F	F	F	F	F	F	F

P = paid historical data

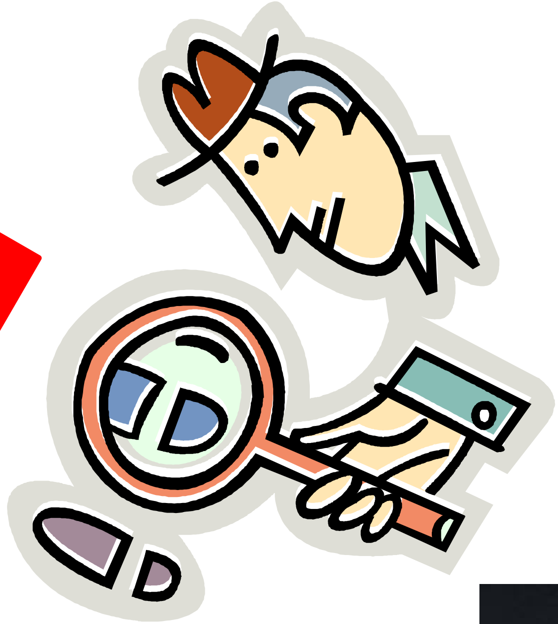
O = outstanding claims for past accident periods

F = future accident period

Premium Liability vs. Accident Period

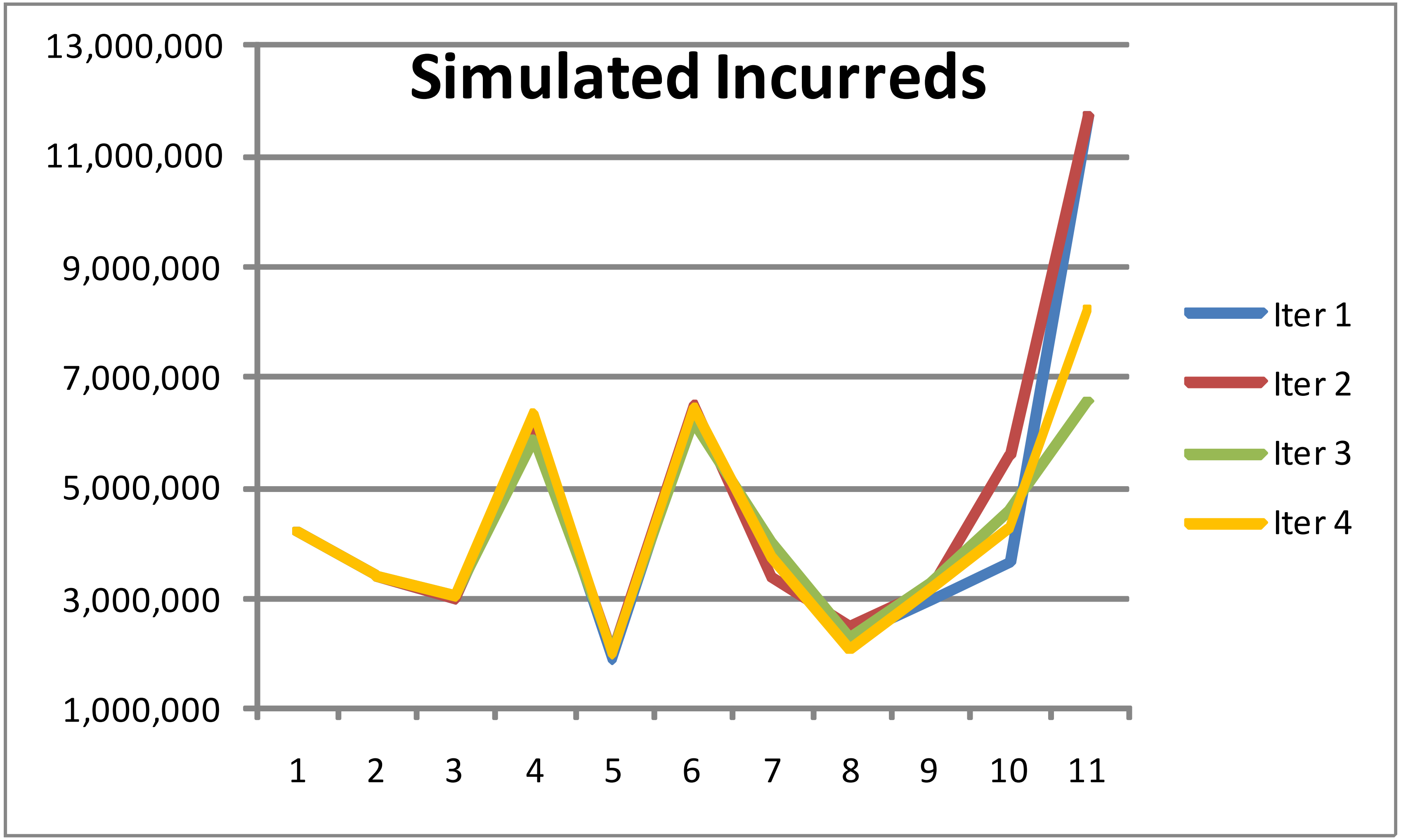


Past -> Future



Past is Unknown

Past is Unknown



Next Accident Period

P	P	P	P	P	P	P	P	P	O
P	P	P	P	P	P	P	P	O	O
P	P	P	P	P	P	P	O	O	O
P	P	P	P	P	P	O	O	O	O
P	P	P	P	P	O	O	O	O	O
P	P	P	P	O	O	O	O	O	O
P	P	P	O	O	O	O	O	O	O
P	P	O	O	O	O	O	O	O	O
P	O	O	O	O	O	O	O	O	O
F	F	F	F	F	F	F	F	F	F

P = paid historical data

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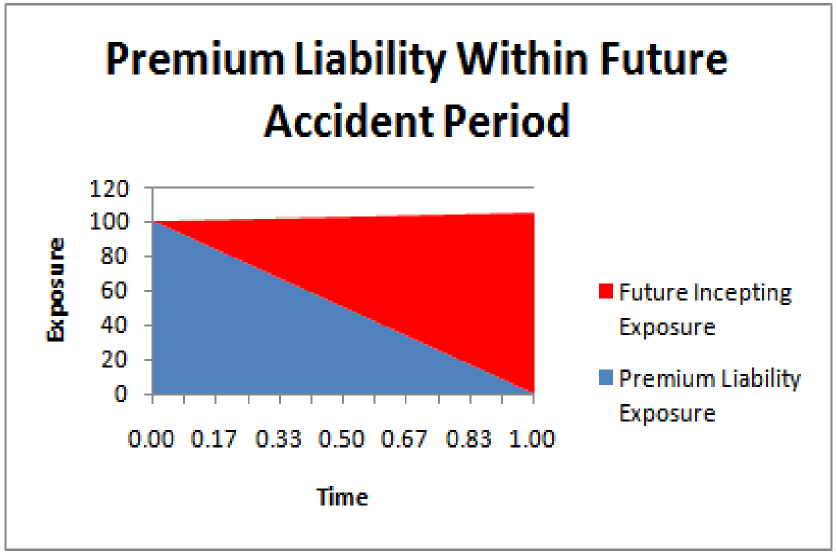
Summary: Step 1

- Estimate of outstanding must be correlated to estimate of next accident period because both are estimated from the same data
- Bootstrap the past accident period variability to estimate the
  - variability of next accident period
  - correlation of the estimate of the next accident period to past

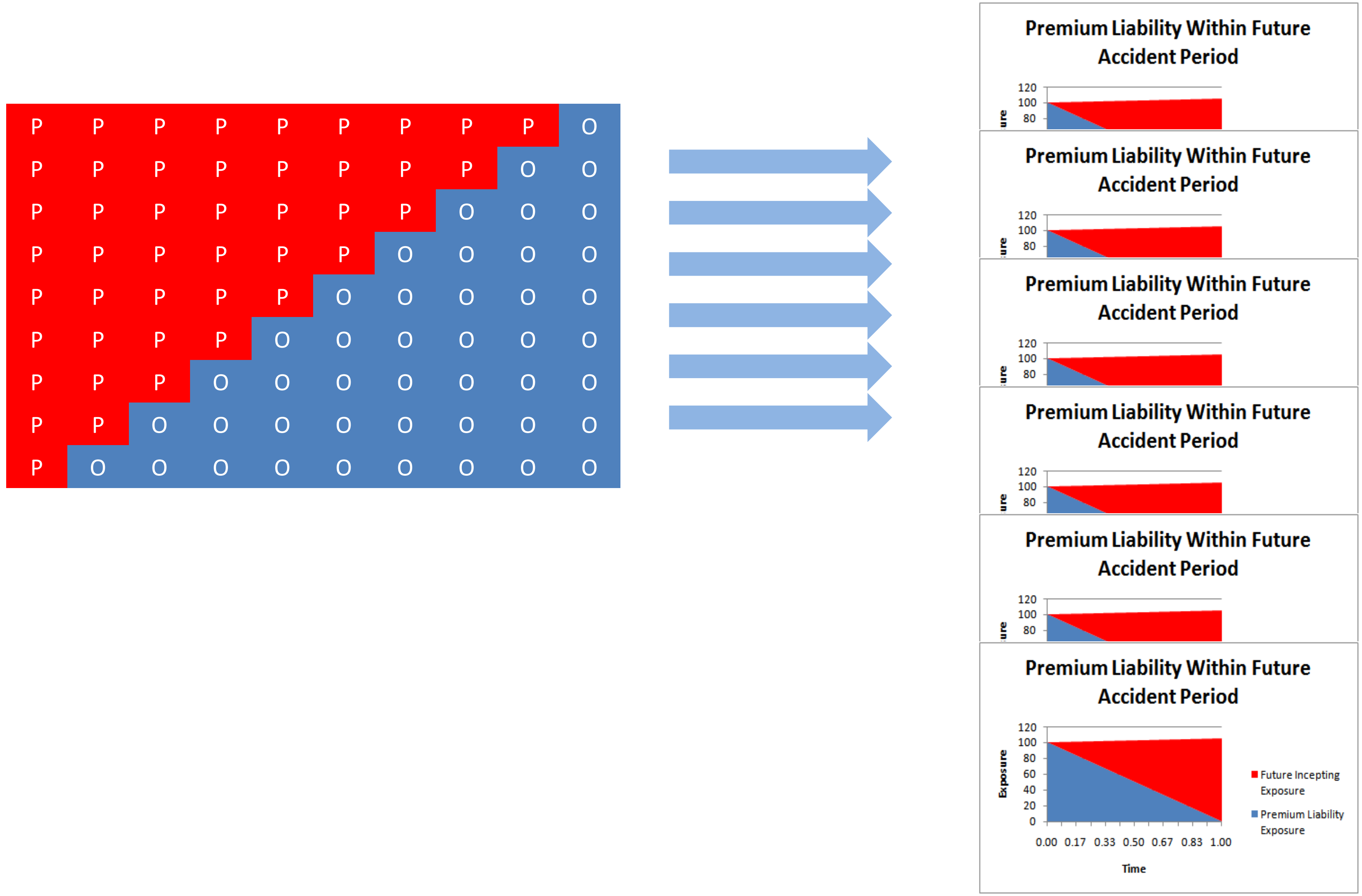
Next Accident Period

P	P	P	P	P	P	P	P	P	O
P	P	P	P	P	P	P	P	O	O
P	P	P	P	P	P	P	O	O	O
P	P	P	P	P	P	O	O	O	O
P	P	P	P	P	O	O	O	O	O
P	P	P	P	O	O	O	O	O	O
P	P	P	O	O	O	O	O	O	O
P	P	O	O	O	O	O	O	O	O
P	O	O	O	O	O	O	O	O	O
F	F	F	F	F	F	F	F	F	F

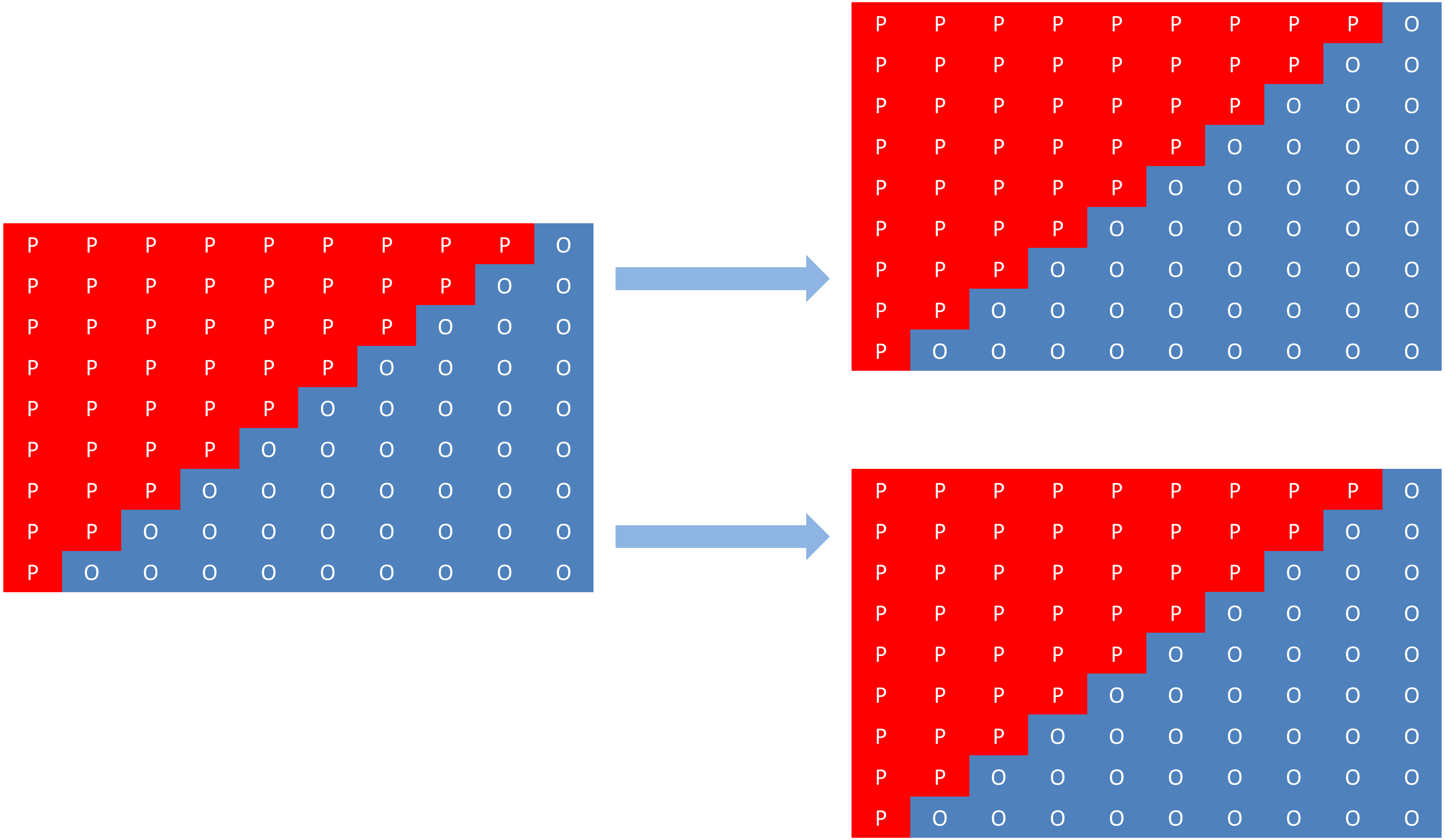
- P = paid historical data
- O = outstanding claims for past accident periods
- F = future accident period



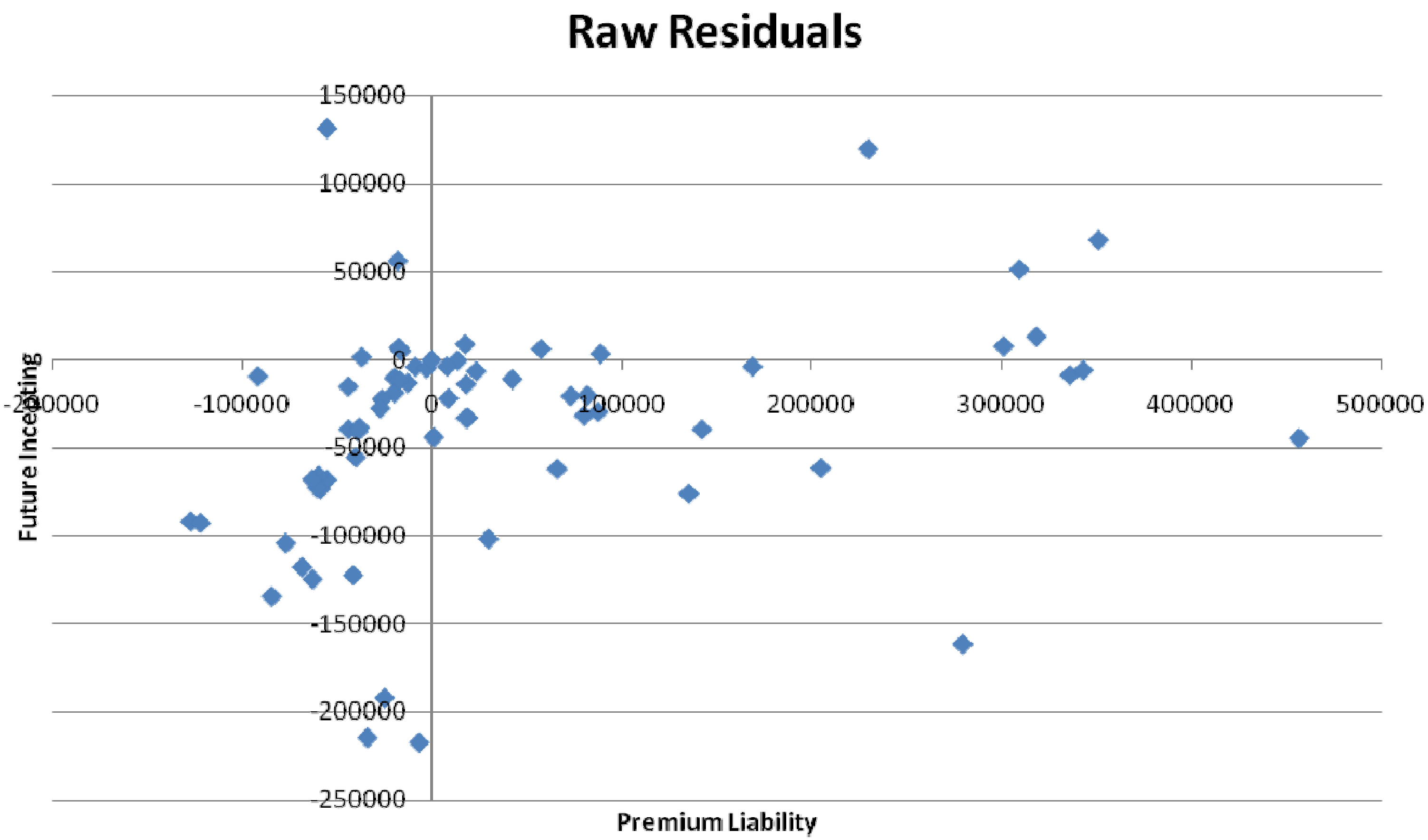
1 Triangle Split in Two



1 Triangle -> 2 Sub-Triangles



Sub-Triangles are Correlated



Detour - Correlations

Simulation - Design

- 2 triangles
- each matching accident period has 80% correlation

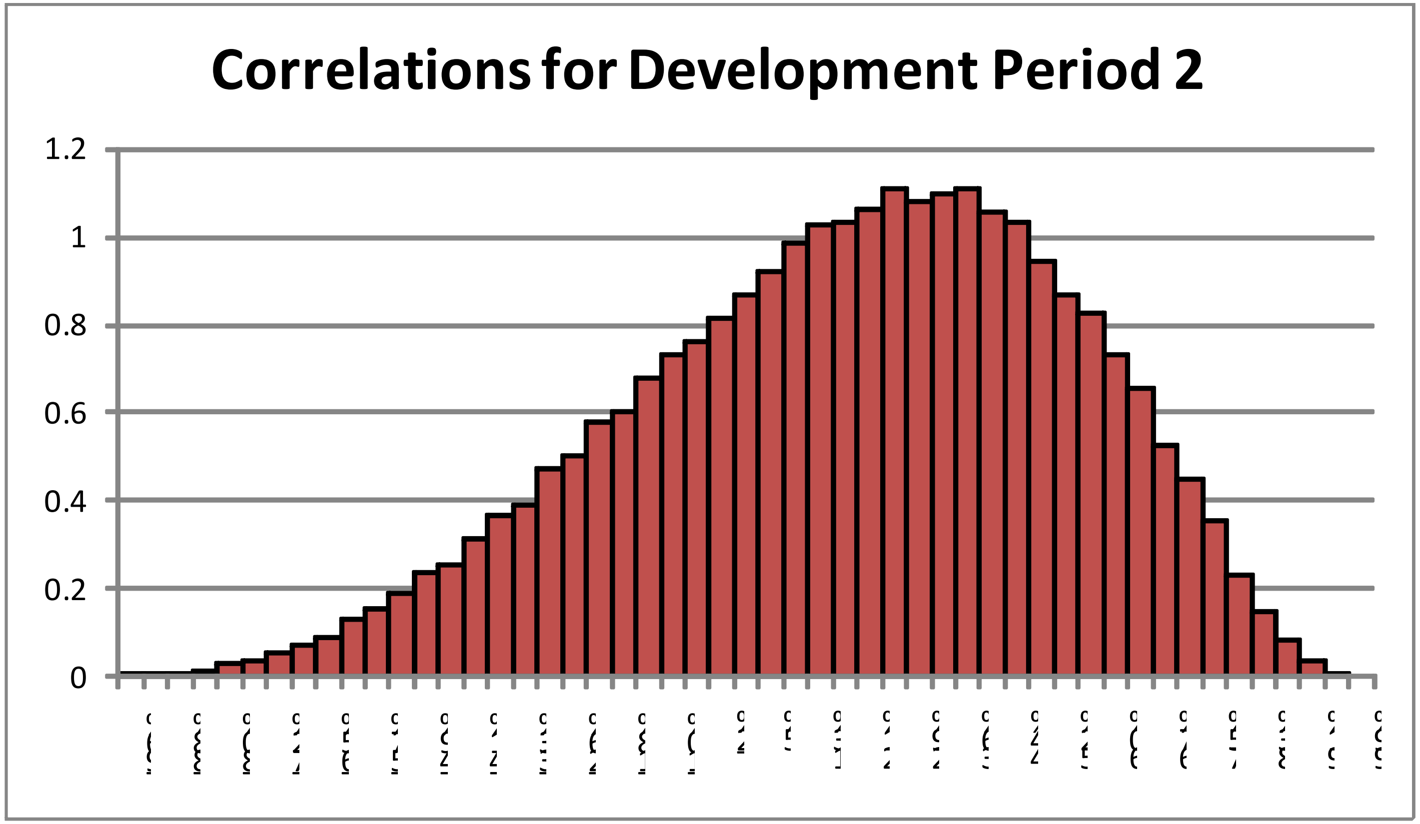
- shouldn’t the triangles then have 80% correlation?

Simulation - Results

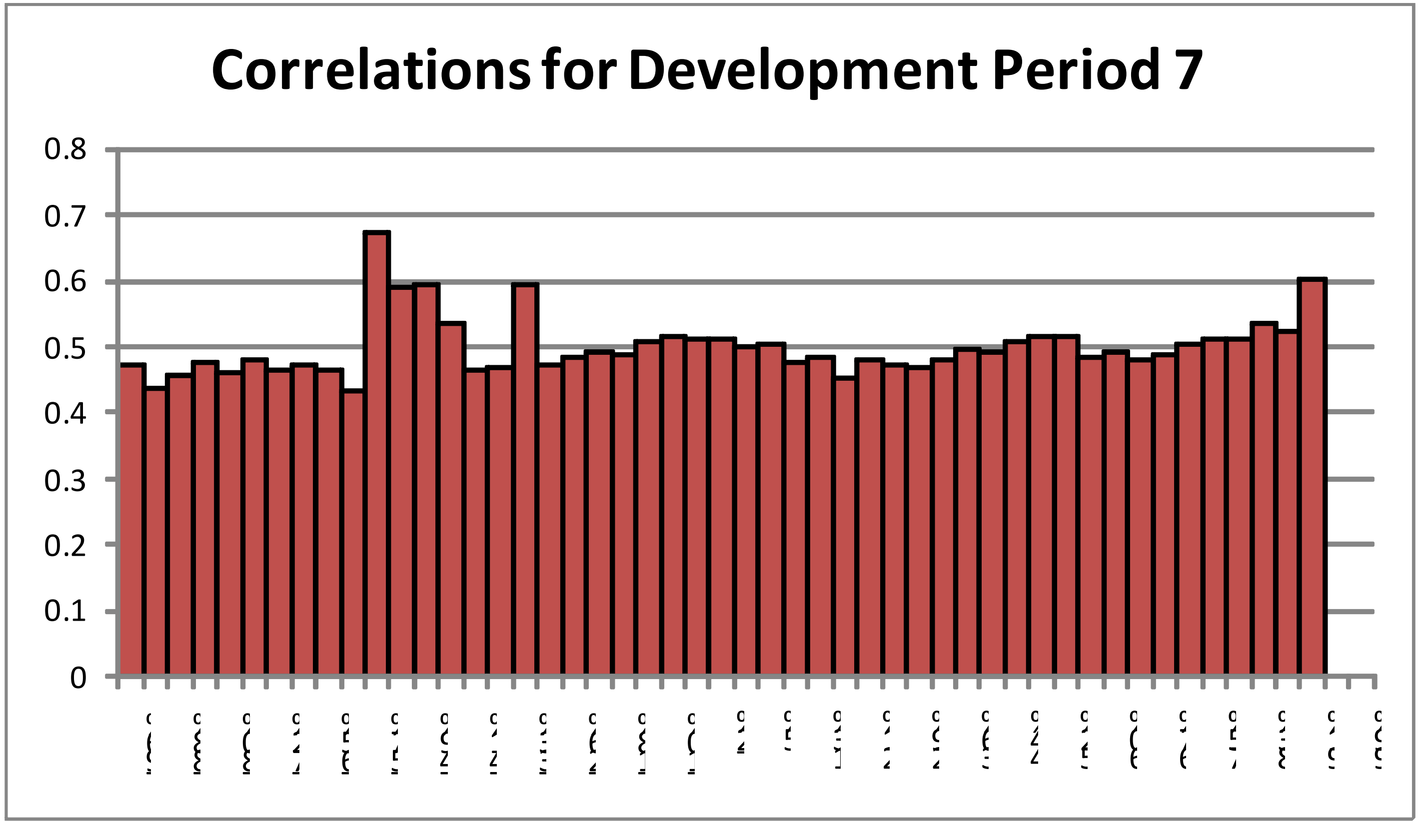
Development Period	Mean Correlation
1	10%
2	20%
3	20%
4	13%
5	6%
6	3%
7	1%
8	0%
9	0%

Huh??????

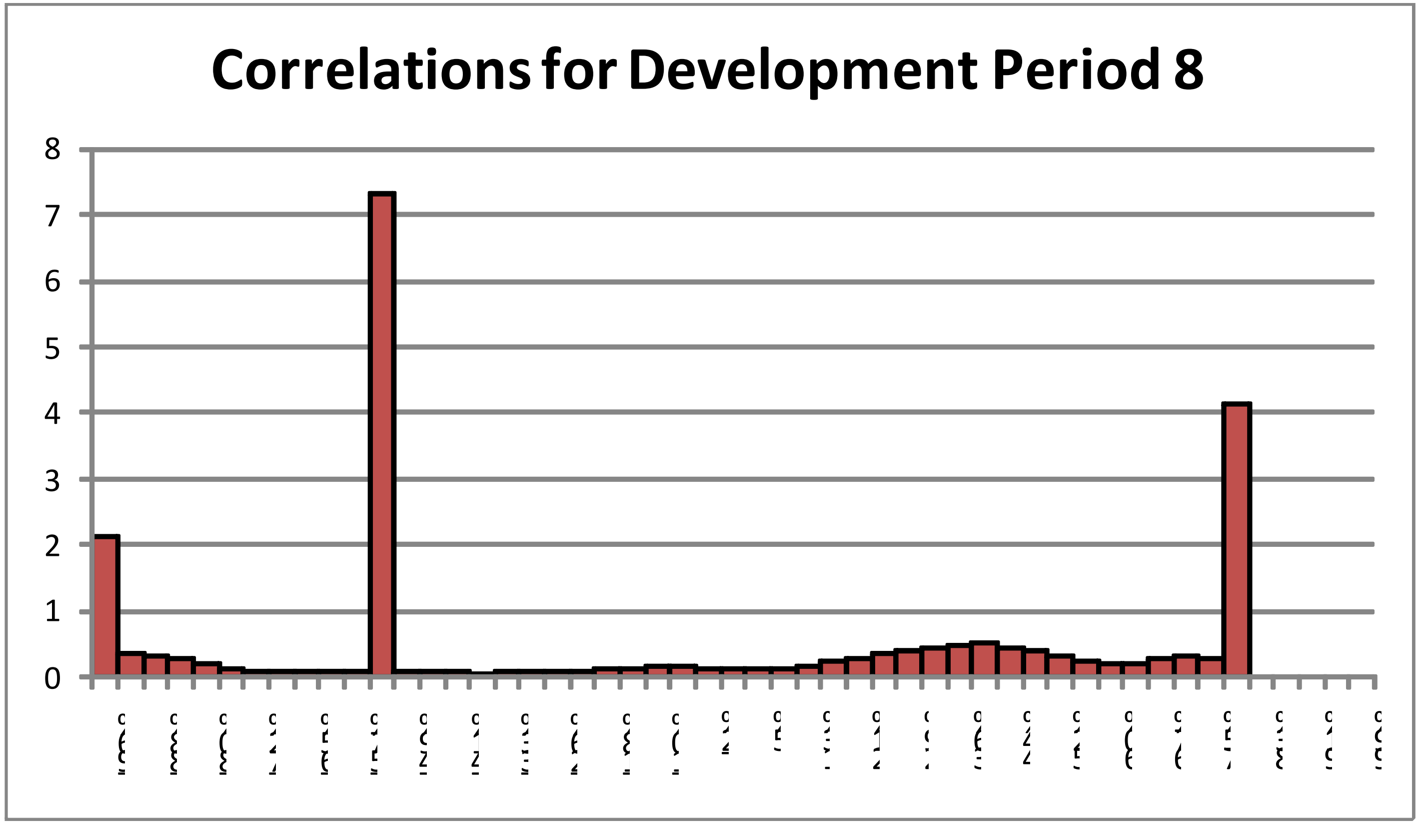
Simulation - Results



Simulation - Results



Simulation - Results

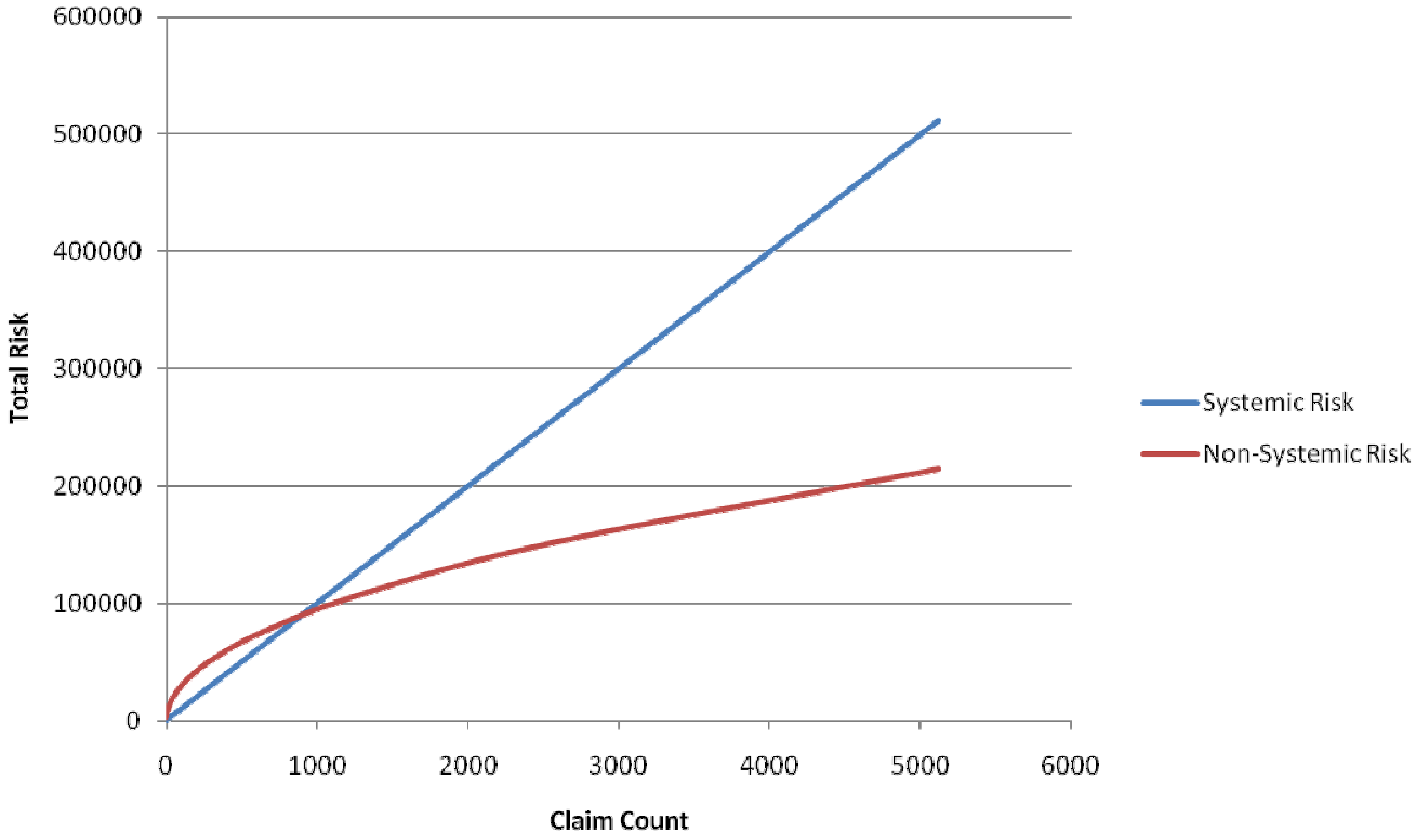


Why Does Correlation Collapse?

- The ratio of systemic risk to non-systemic risk is reducing as we move to higher development periods (because fewer claims)

- BUT this means that we can estimate systemic risk and non-systemic risk

Systemic vs. Non-Systemic



Modelling Systemic vs. Non-Systemic

- fitting a model such that $\sigma(r_{ij}) = \hat{P}_{ij}\alpha^2\beta + \sqrt{\hat{P}_{ij}}\alpha\gamma$
- the maximum likelihood estimate requires us to minimise this function

$$f(\alpha,\beta,\gamma) = \frac{1}{2}n\ln(2\pi) + \sum \ln\left(\hat{P}_{ij}\alpha^2\beta + \sqrt{\hat{P}_{ij}}\alpha\gamma\right) + \frac{1}{2}\sum \frac{R_{ij}^2}{\left(\hat{P}_{ij}\alpha^2\beta + \sqrt{\hat{P}_{ij}}\alpha\gamma\right)^2}$$

- this means that the correlation of the two sub-triangles is estimated as

$$\rho = \frac{\alpha n_A}{\sqrt{\alpha^2 n_A^2 + \beta^2 n_A}} \times \frac{\alpha n_B}{\sqrt{\alpha^2 n_B^2 + \beta^2 n_B}}$$

Back On Track

- The estimate of the standard deviation of the premium liability can be expressed as a function of the standard deviation of the whole accident period (which we estimated earlier in this presentation)

$$\sigma_A = \frac{\sqrt{\alpha^2 n_A^2 + \beta^2 n_A}}{\sqrt{\alpha^2 (n_A + n_B)^2 + \beta^2 (n_A + n_B)}} \sigma_{A+B}$$

Summary: Step 2

- Fit a model to separate systemic from non-systemic risk
- Use the model parameters to obtain an estimate for the standard deviation of the premium liability

More Algebra!

- correlation of premium liability and the whole accident period is

$$\frac{\alpha^2 n_A^2 + \alpha^2 n_A n_B + \beta^2 n_A}{\sqrt{\alpha^2 n_A^2 + \beta^2 n_A}\sqrt{(n_A + n_B)\left(\beta^2 + \alpha^2 n_A + \alpha^2 n_B\right)}}$$

Summary: Step 3

- combine the correlations of
  - outstanding claims and next accident period
  - next accident period and premium liability
- it is reasonable to assume that these relationships are independent

Summary

- the way to estimate correlations is to estimate systemic and non-systemic risk