

LONGEVITY RISK AND MODELS FOR FORECASTING TRENDS IN MORTALITY RATES

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June 26th 2012

AGENDA FOR TALK

- Background to the longevity problem
- Measuring and quantifying longevity risk
- Comparative study of performance of stochastic mortality dynamics models
- Reflections on results

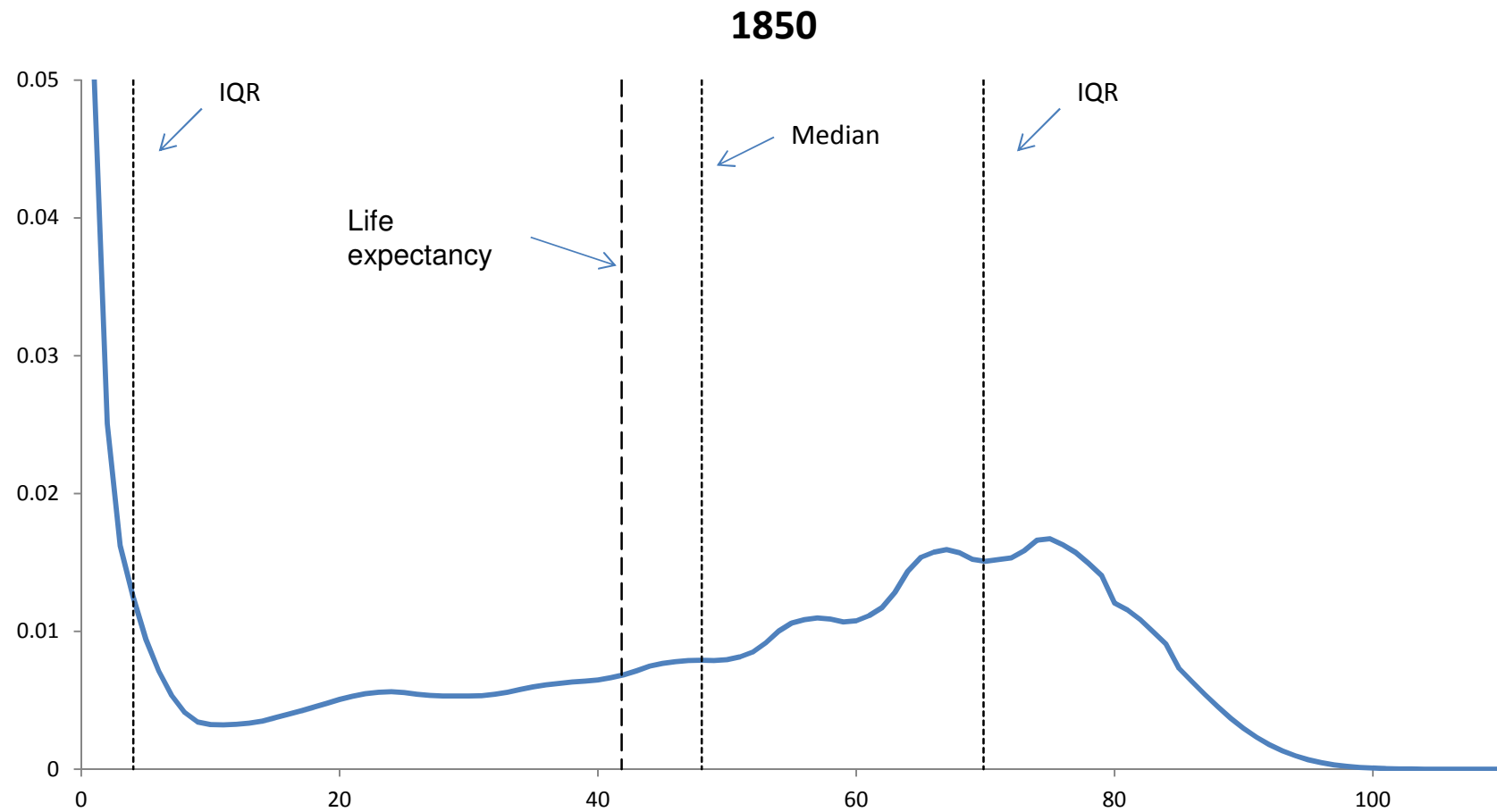
BACKGROUND TO THE LONGEVITY PROBLEM

- What is Longevity Risk?
 - Average lifetime for those at adult/old ages is longer than what is expected
 - Systematic effect
 - Causes include misspecification of mortality model, biased estimation of parameters

MAIN DEMOGRAPHIC FEATURES

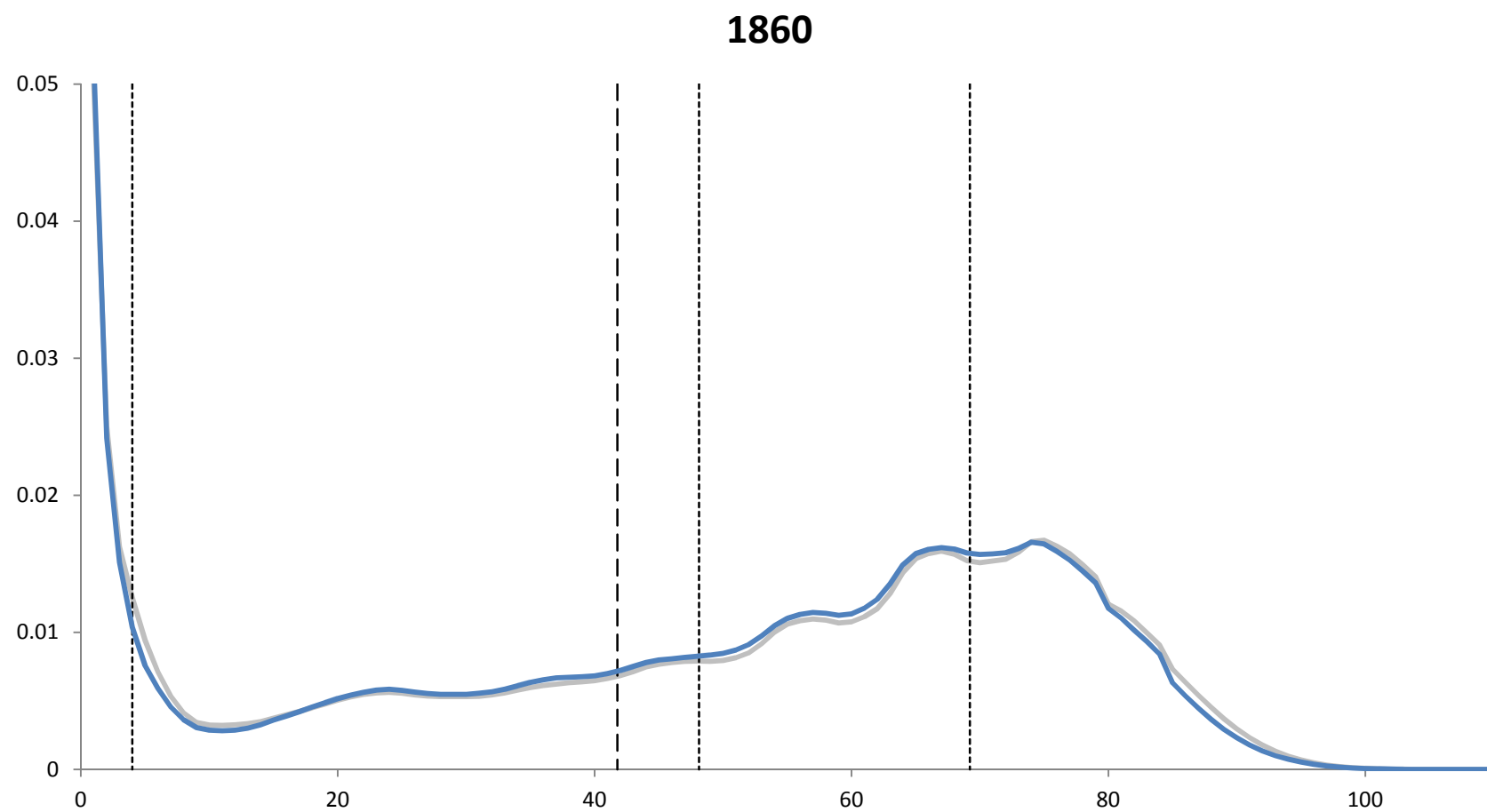
- Expansion over time
- Rectangularization over time
- Downward trend over time in death rates
- Increasing trend over time in life expectancy

Male life Table distribution of deaths, England and Wales 1850-2009



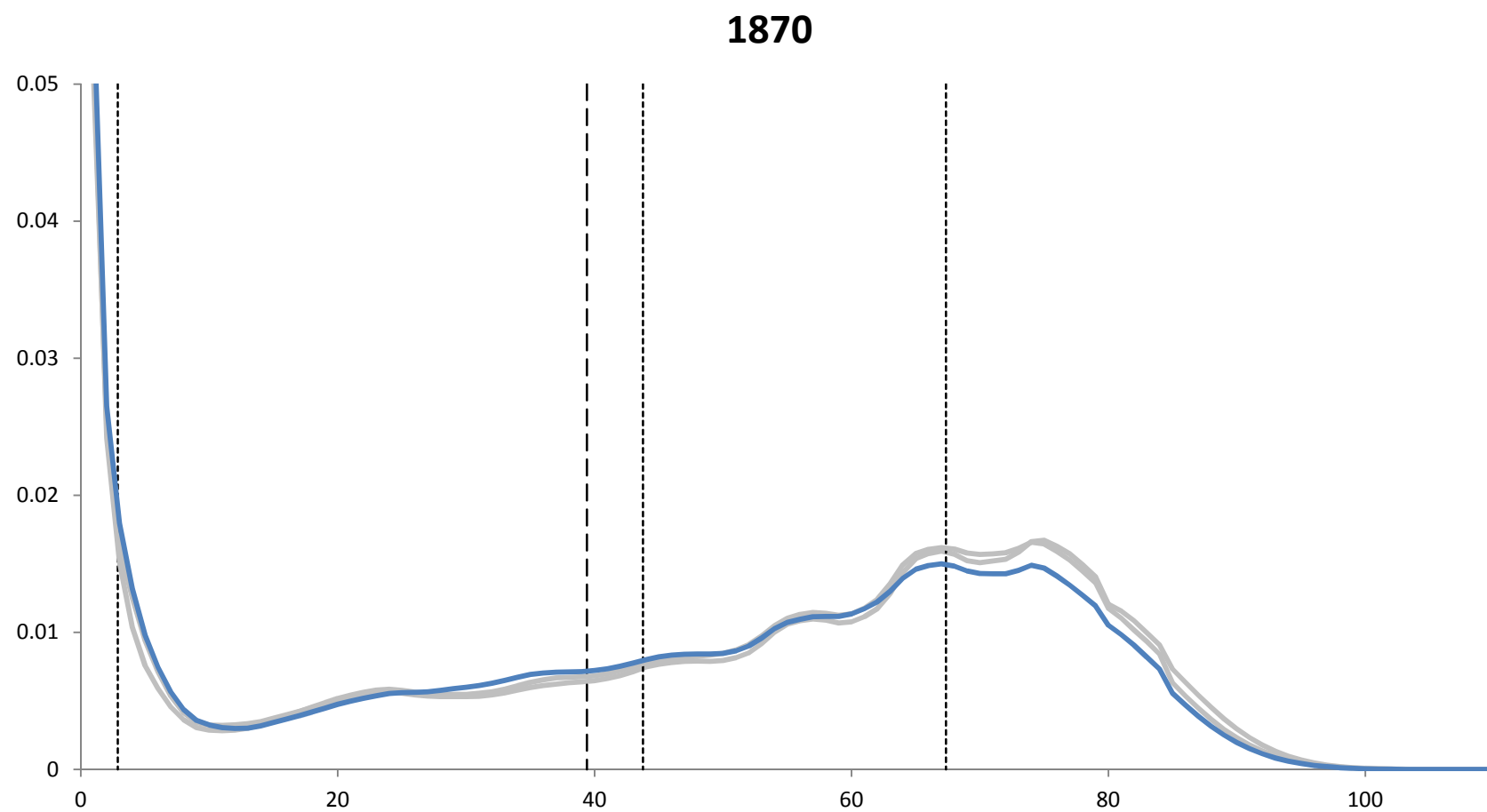
Source: Human Mortality Database

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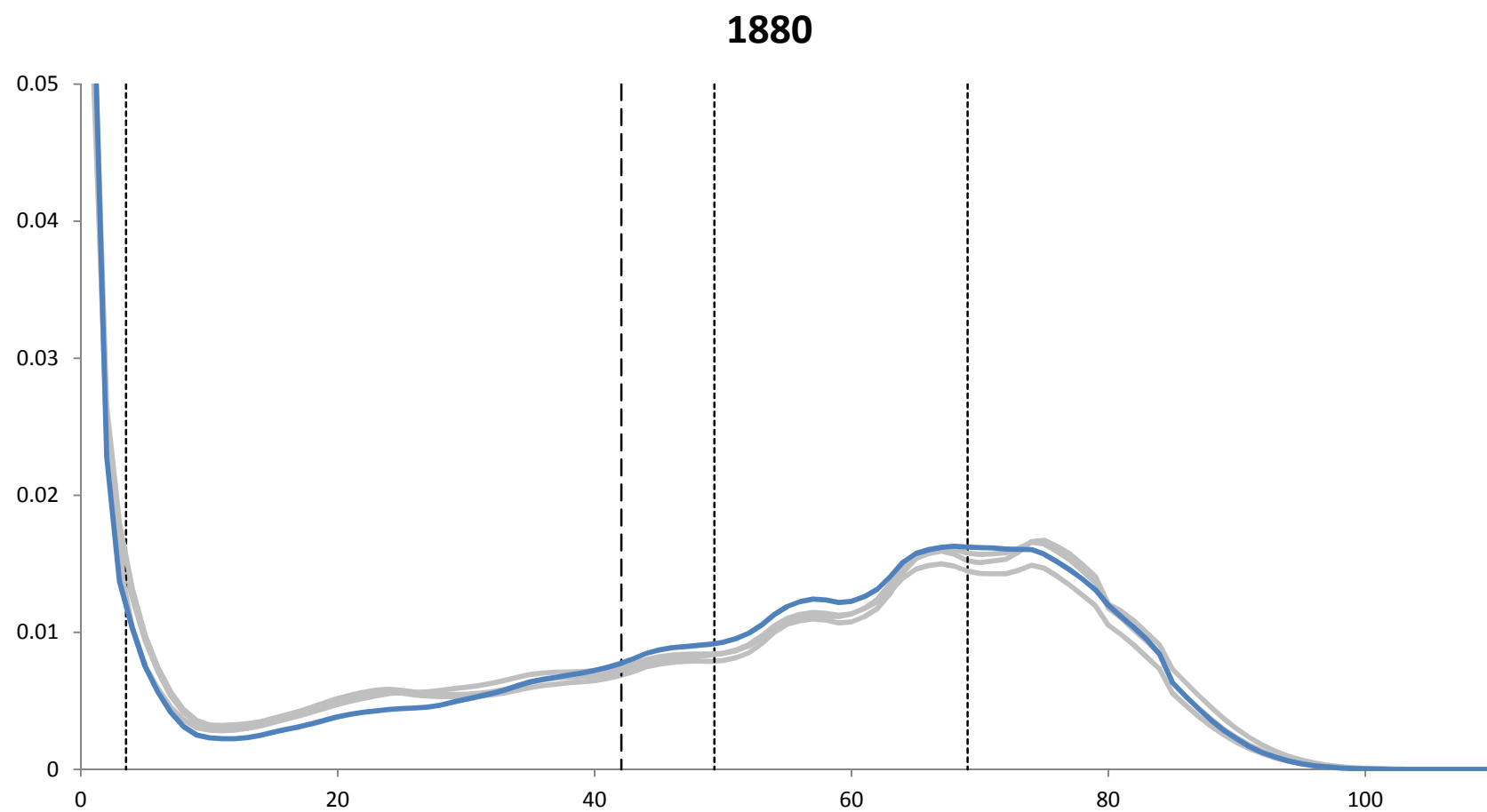
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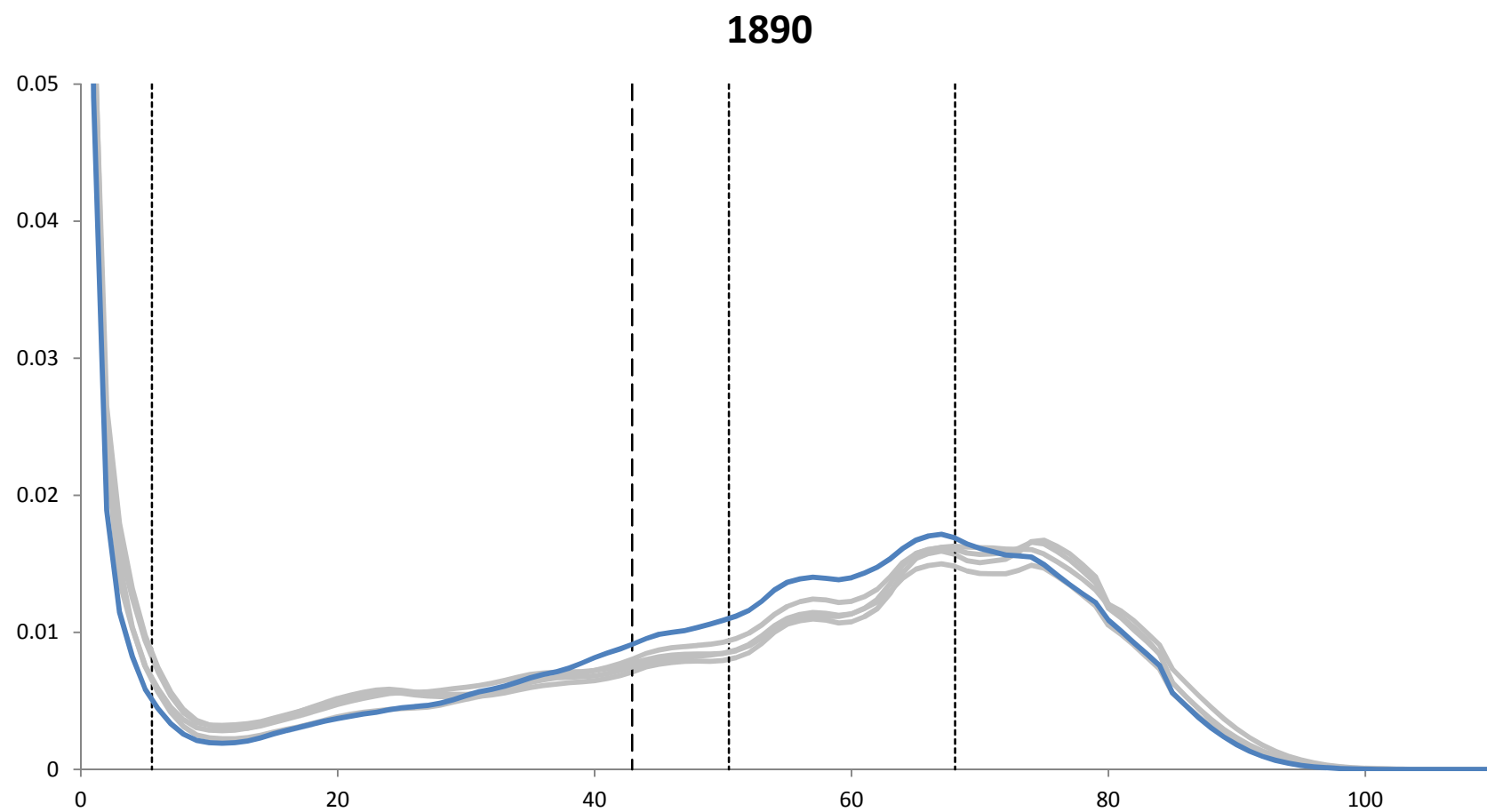
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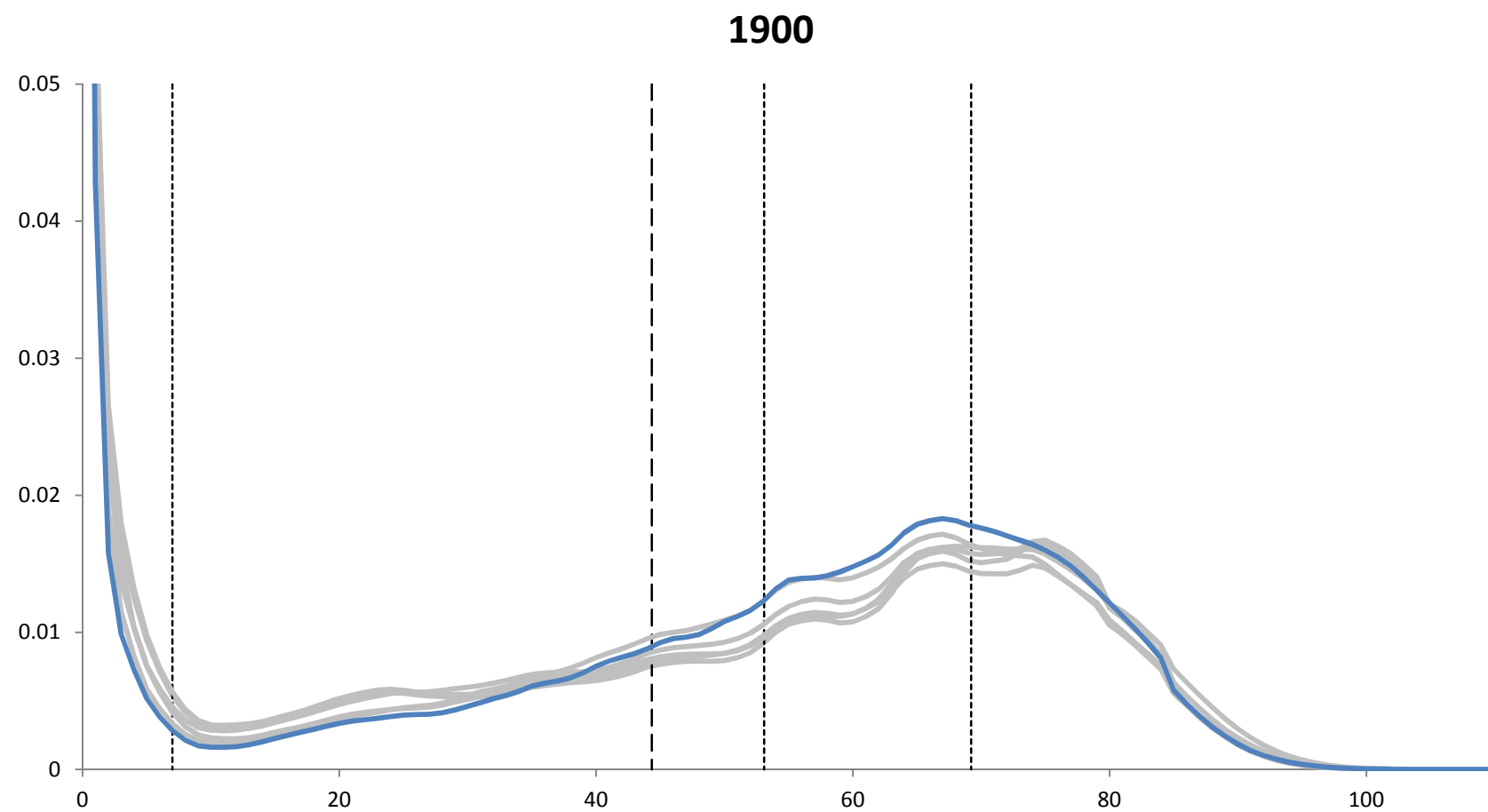
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Male life Table distribution of deaths, England and Wales 1850-2009



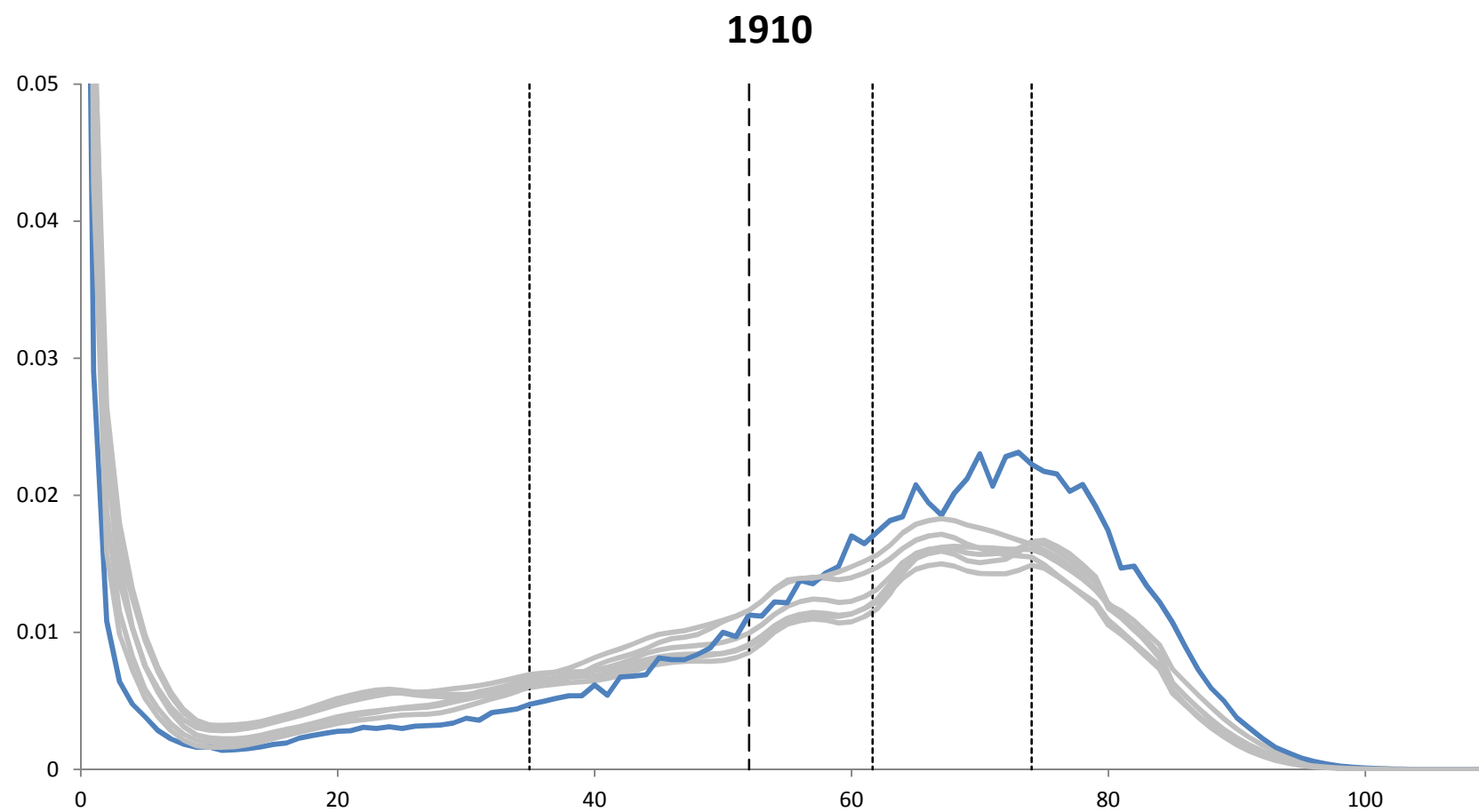
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Male life Table distribution of deaths, England and Wales 1850-2009



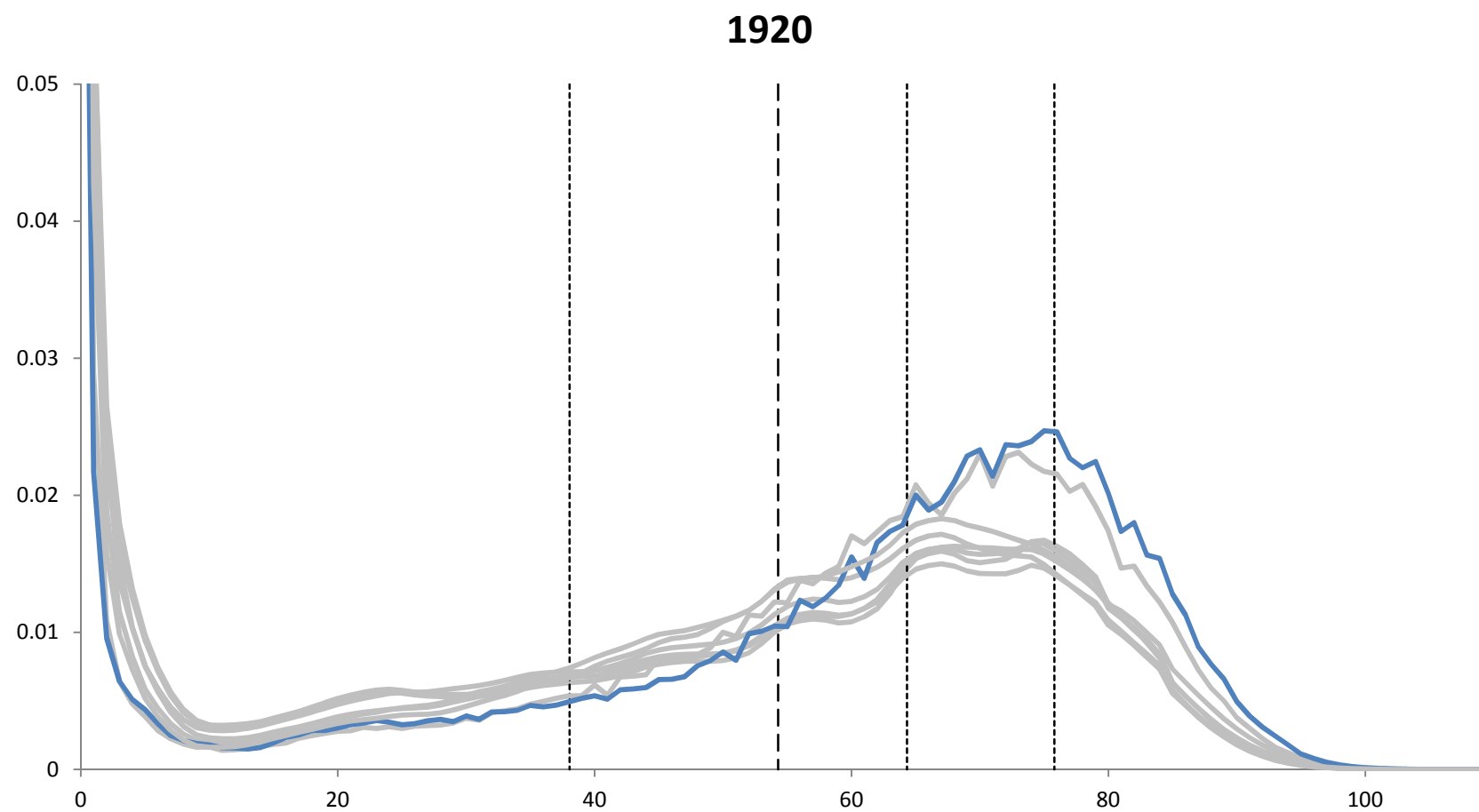
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Male life Table distribution of deaths, England and Wales 1850-2009



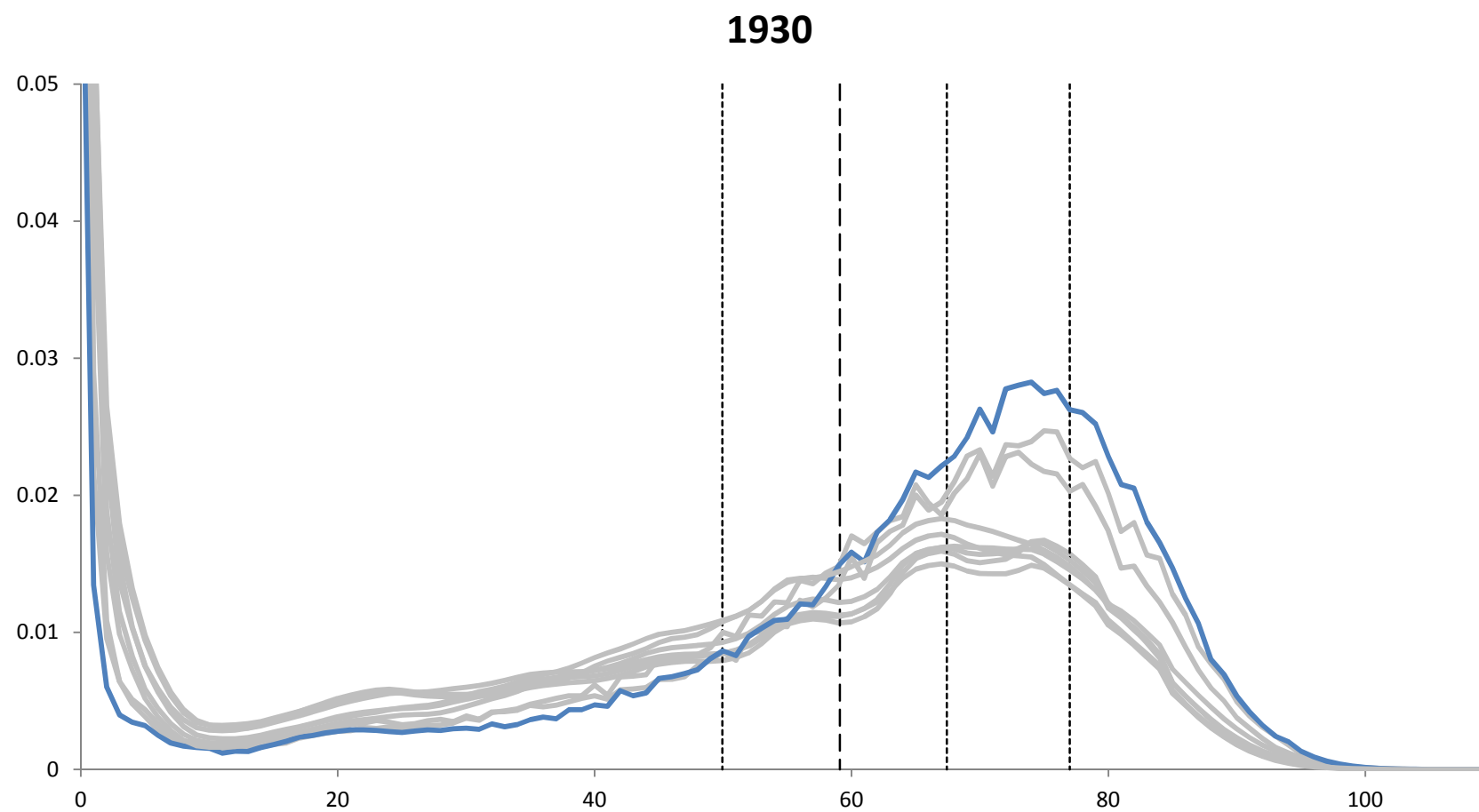
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Male life Table distribution of deaths, England and Wales 1850-2009



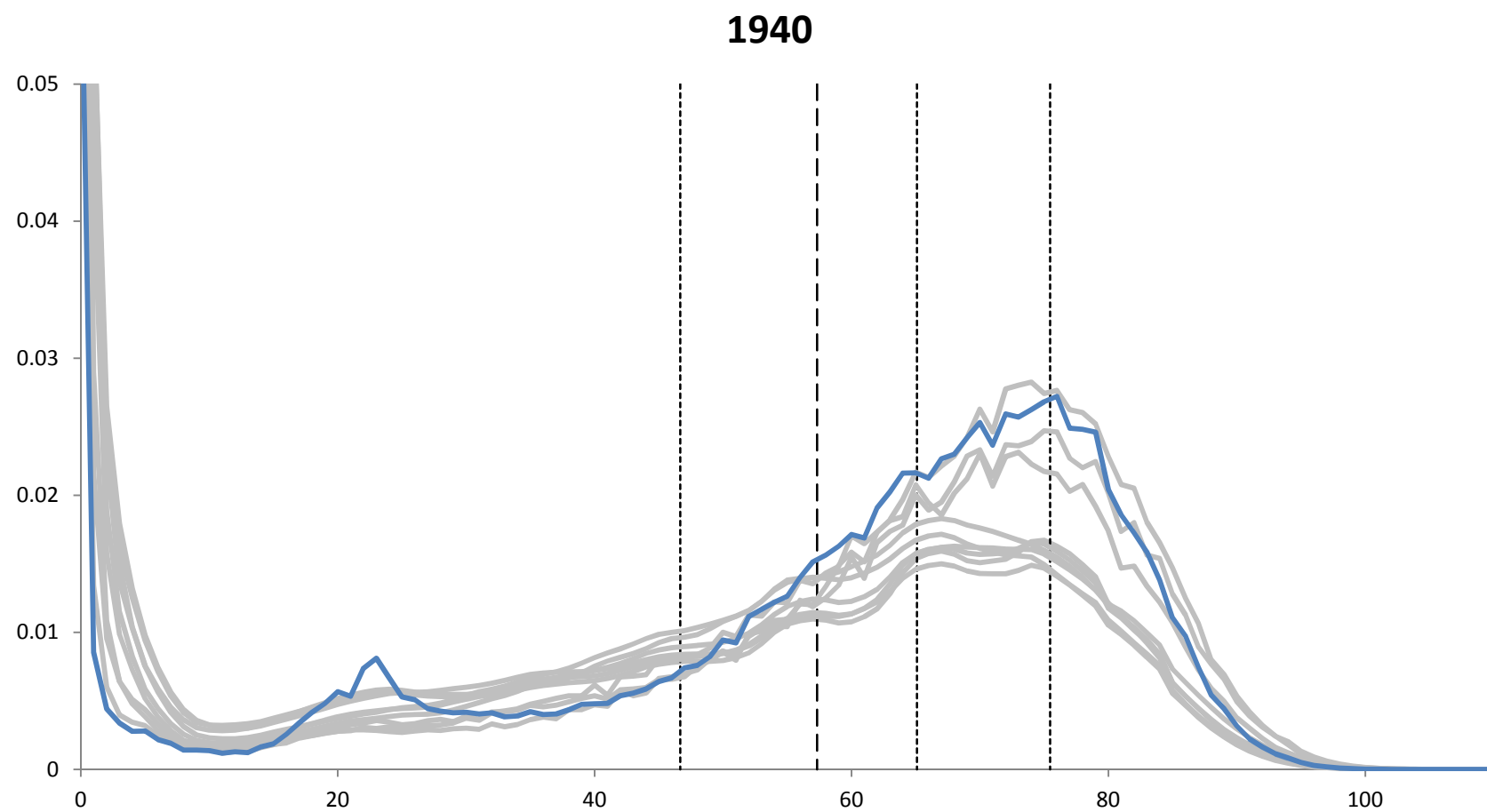
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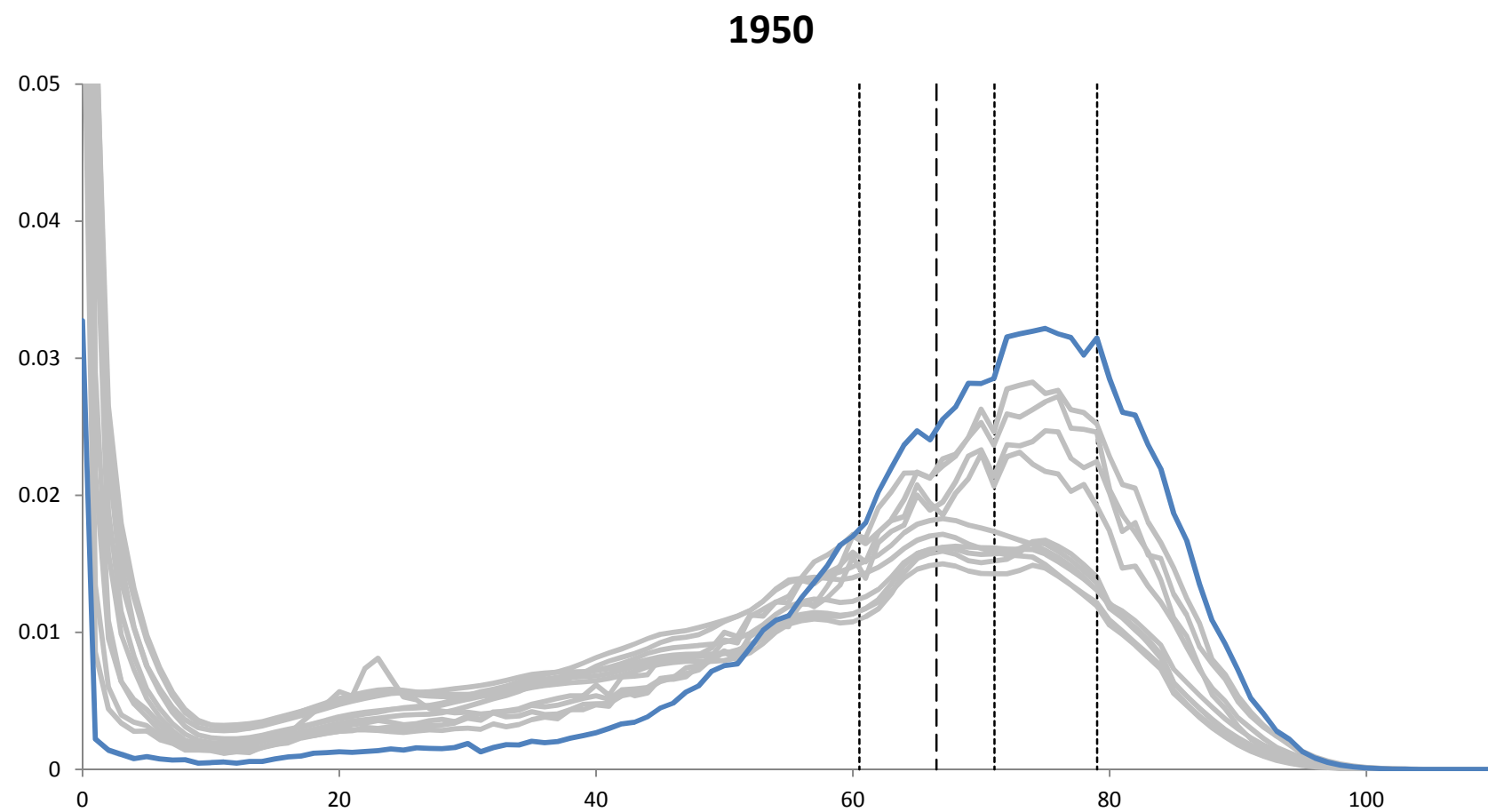
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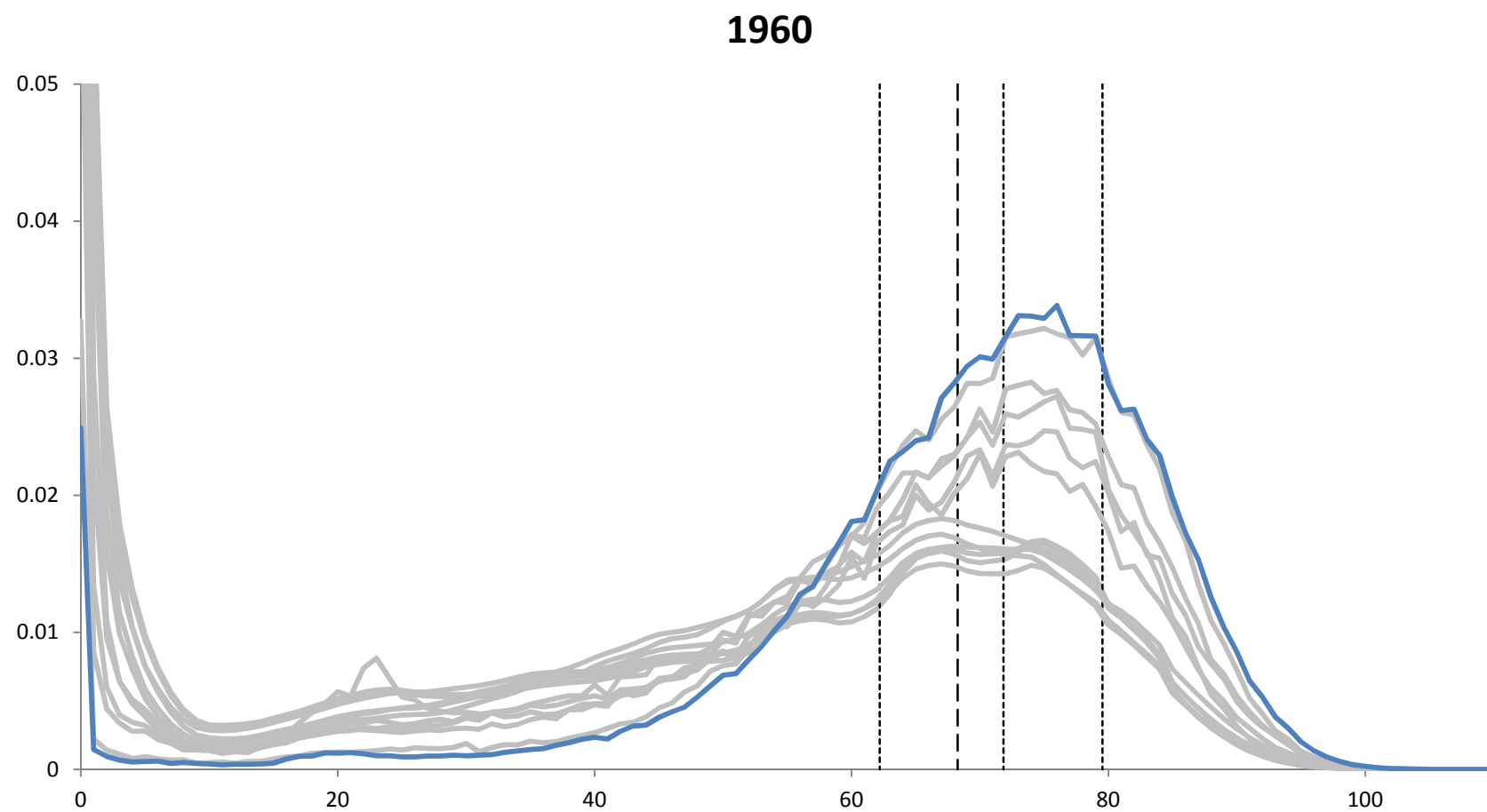
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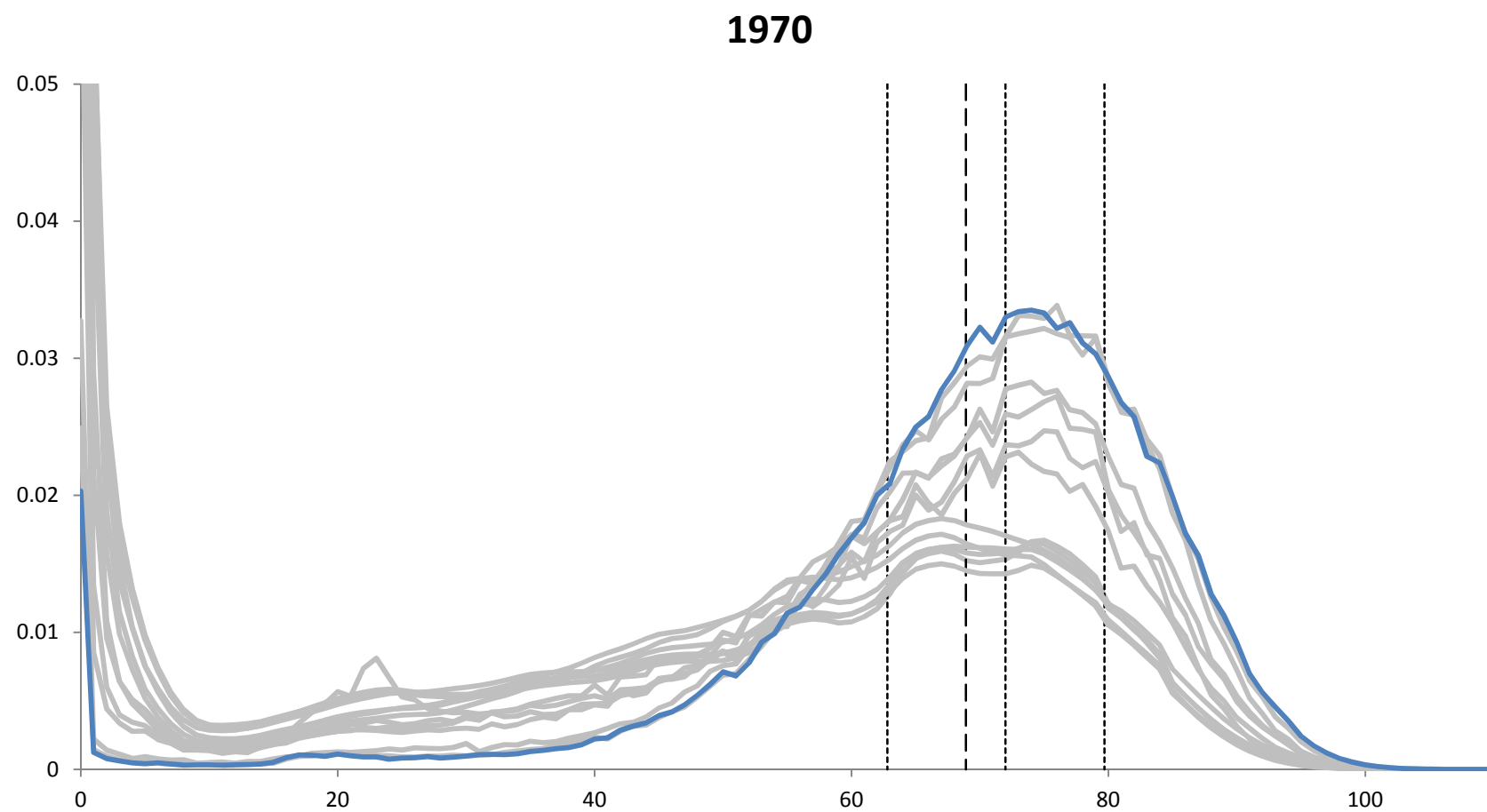
Source: Human Mortality Database

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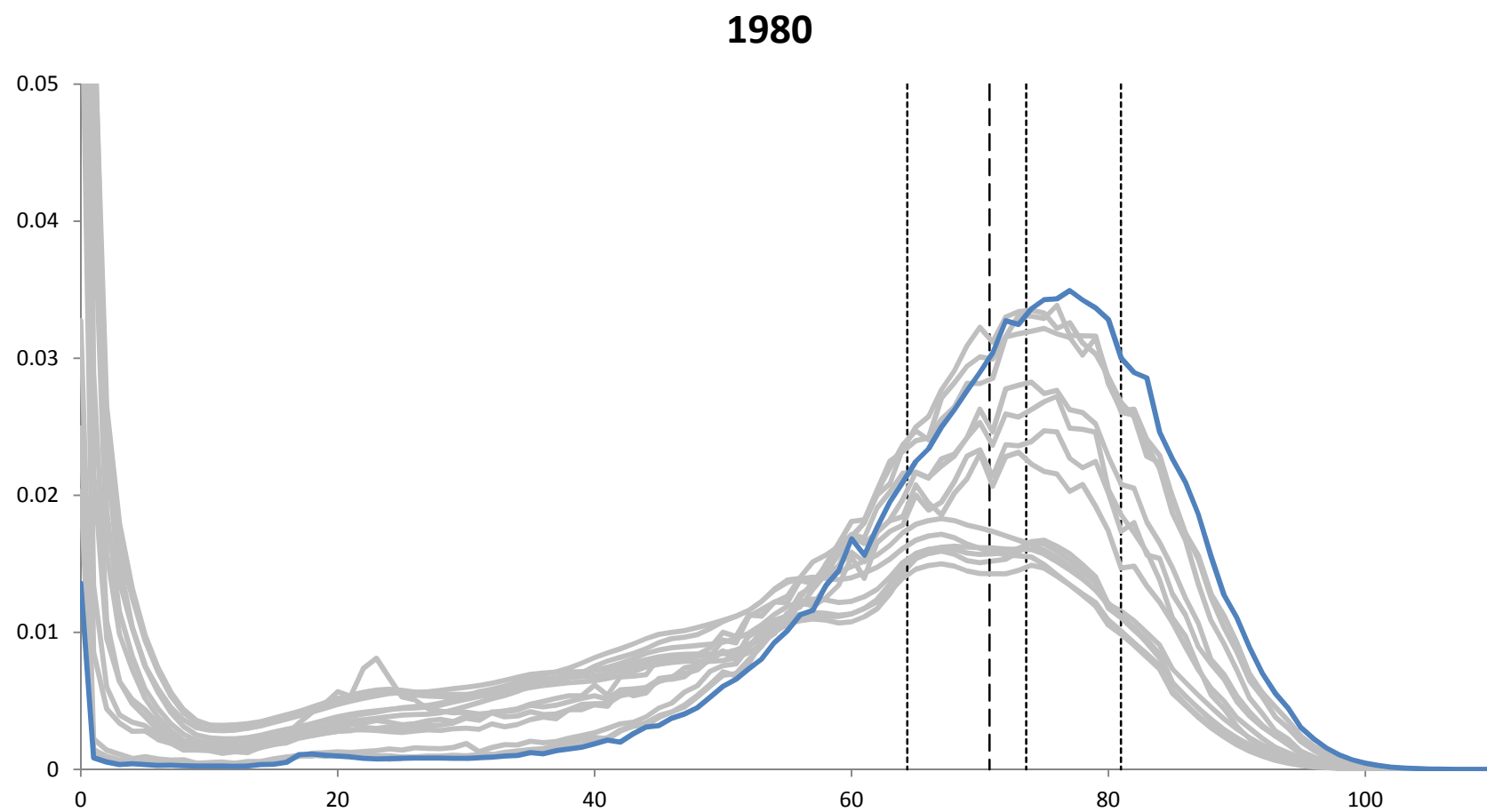
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Male life Table distribution of deaths, England and Wales 1850-2009



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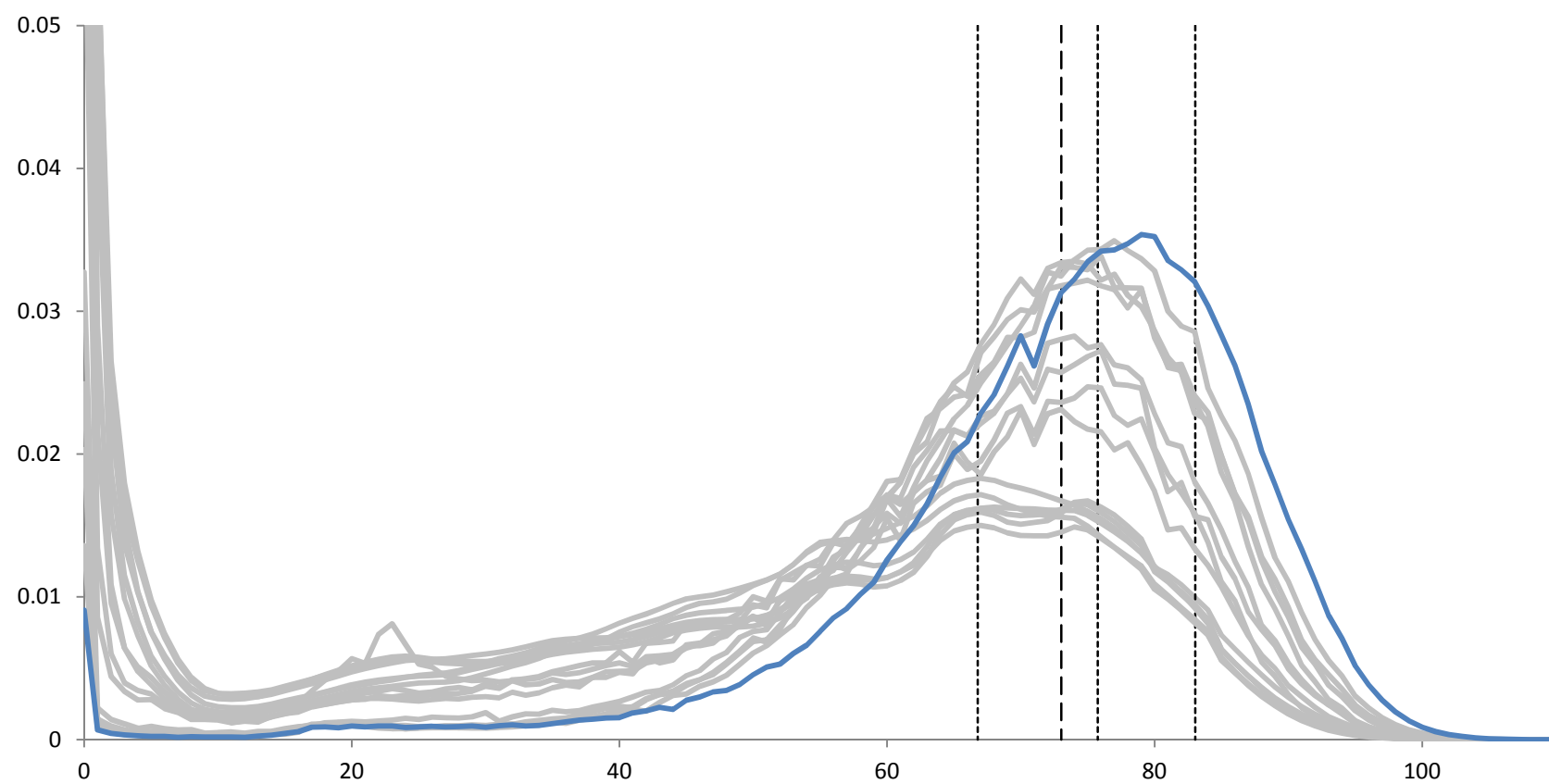
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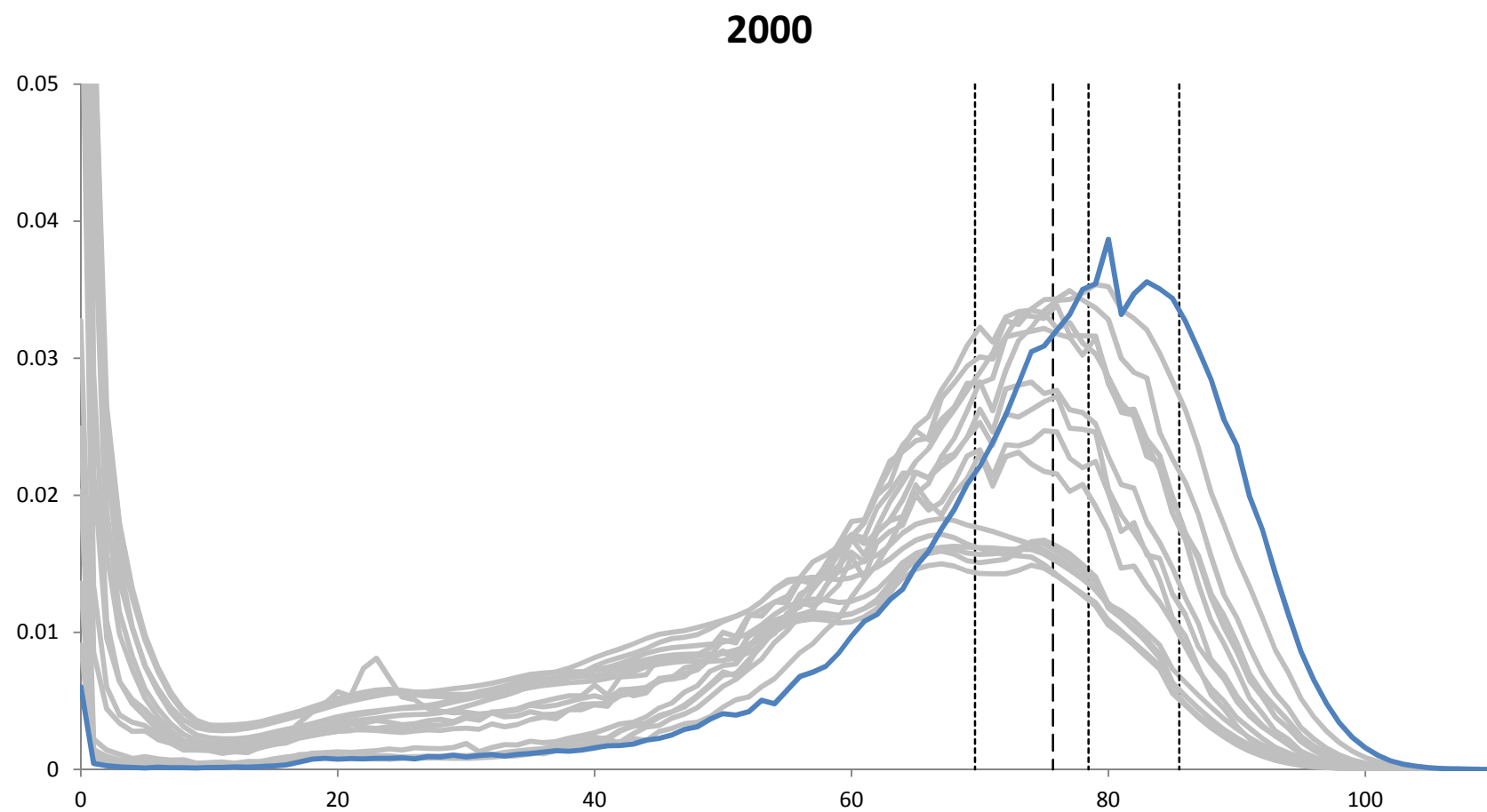
Male life Table distribution of deaths, England and Wales 1850-2009

1990



Source: Human Mortality Database

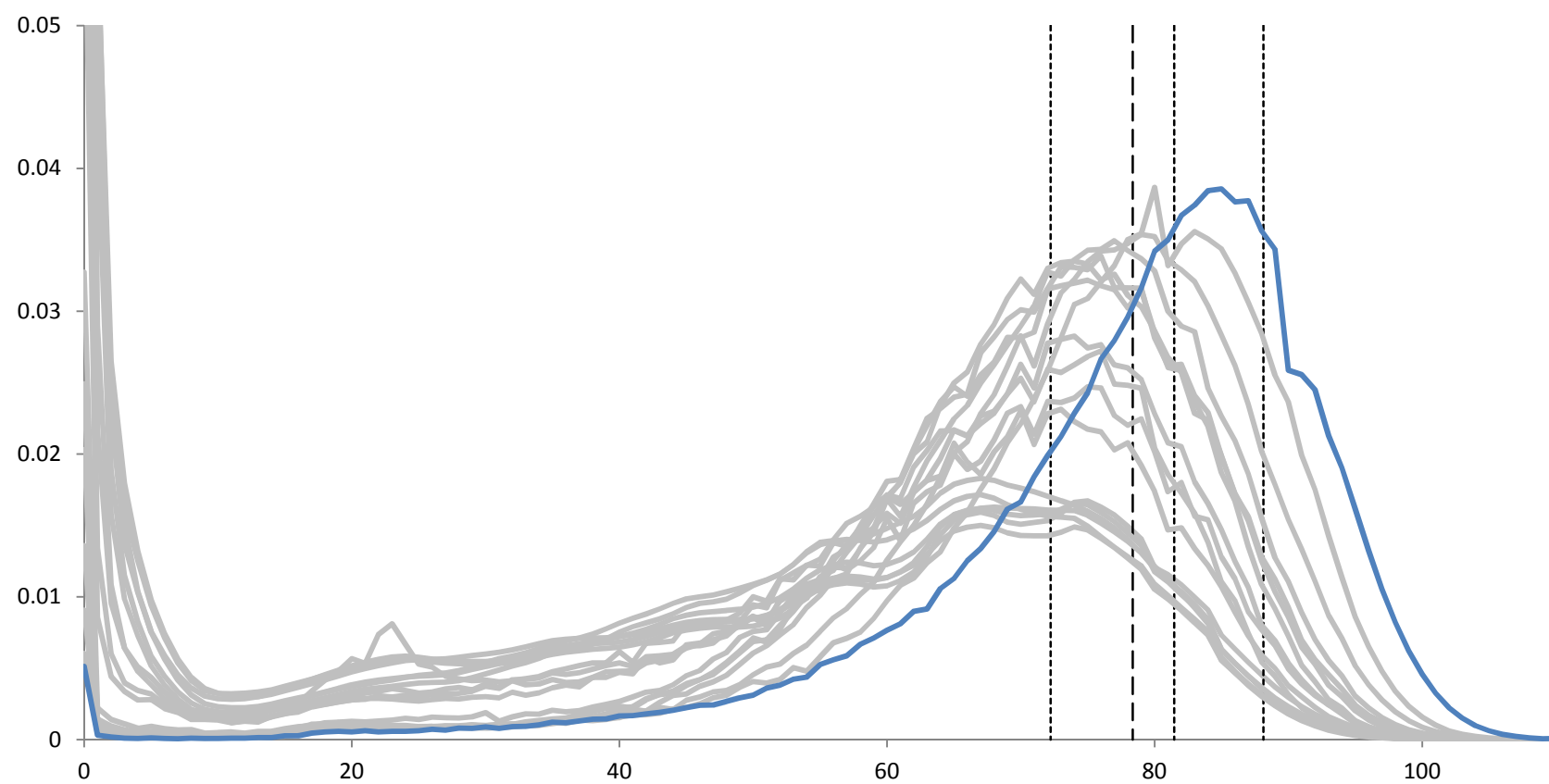
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Source: Human Mortality Database

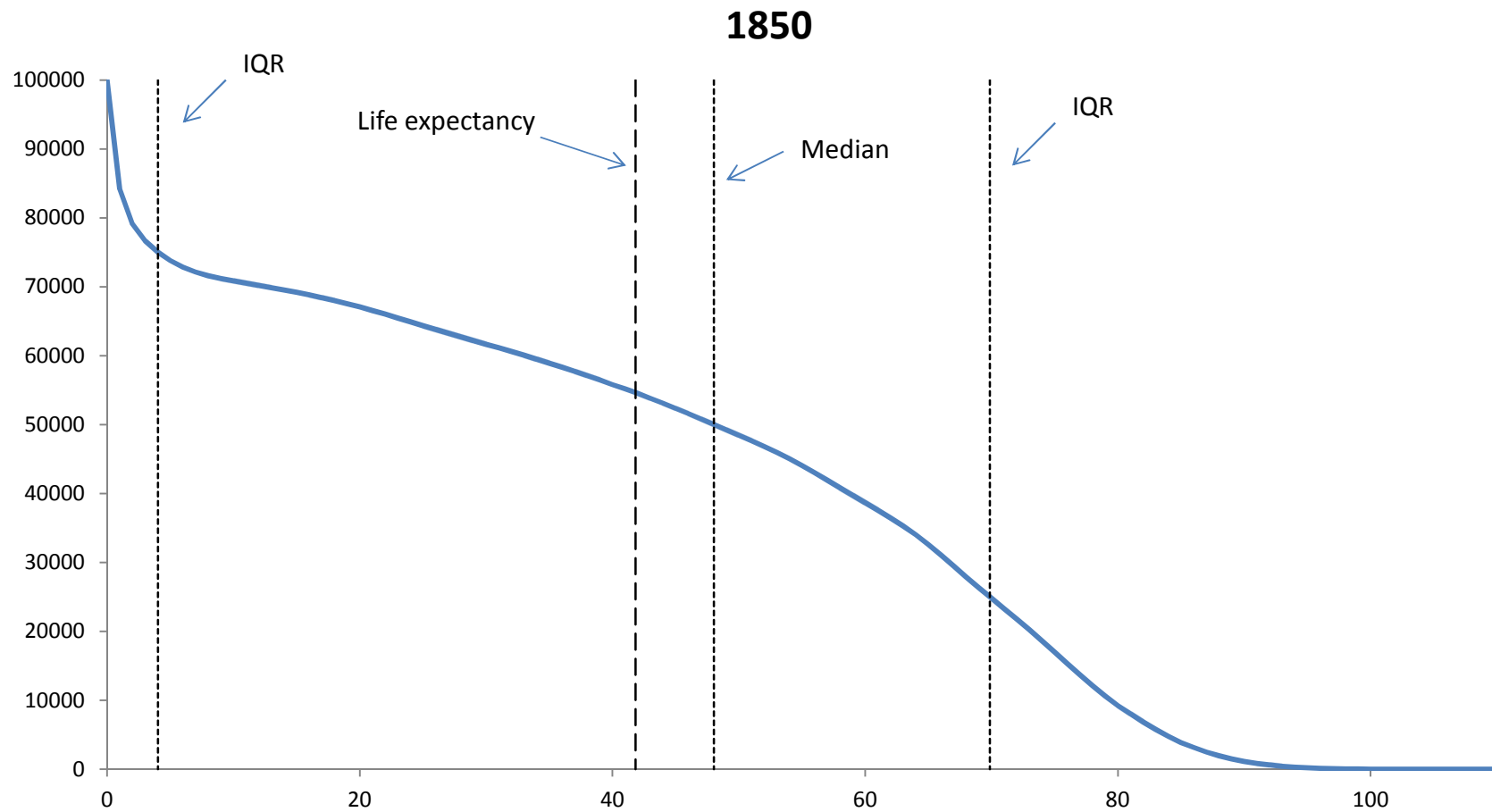
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2009



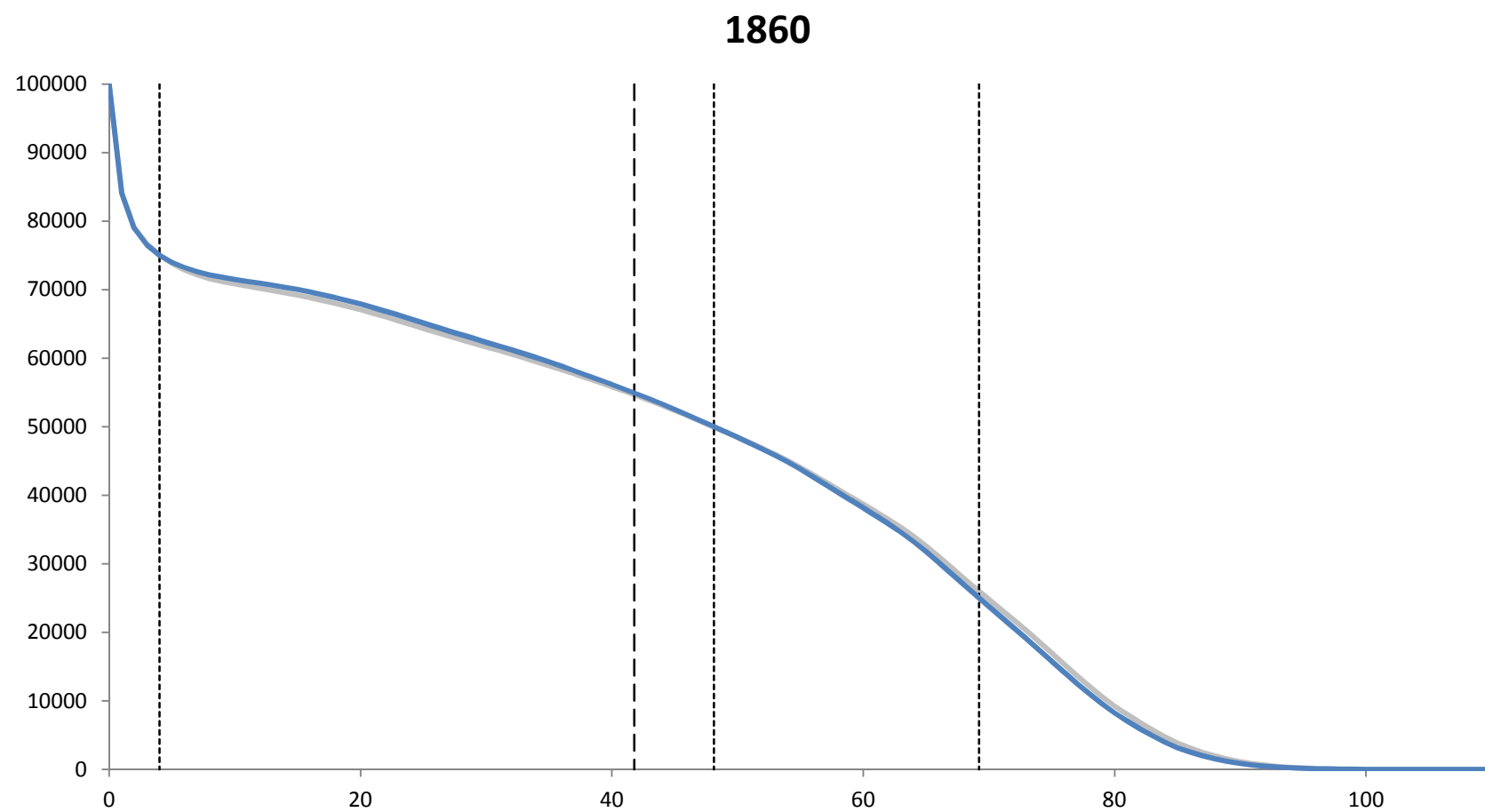
Source: Human Mortality Database

Male number of survivors, England and Wales 1850-2009



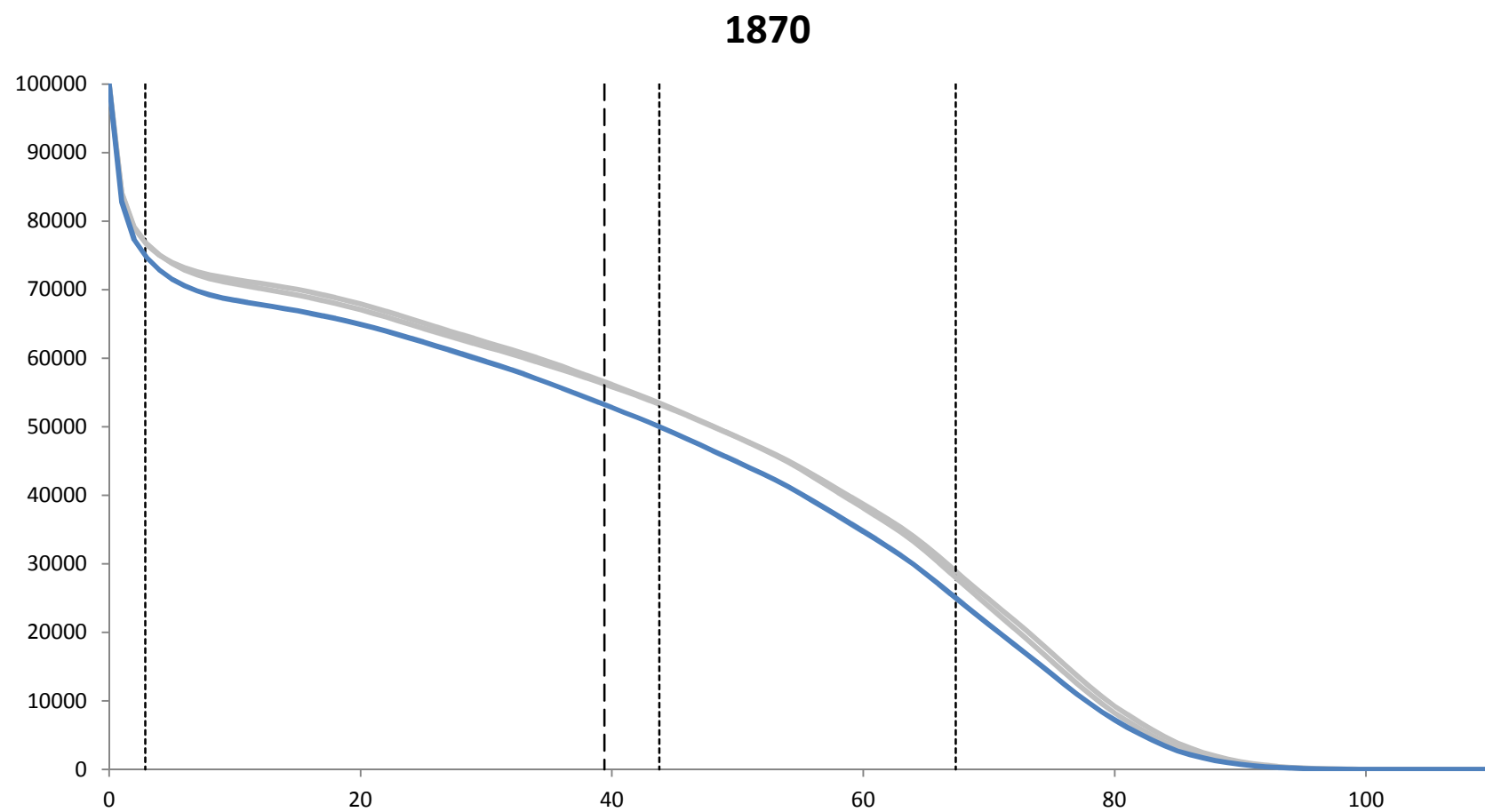
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Male number of survivors, England and Wales 1850-2009



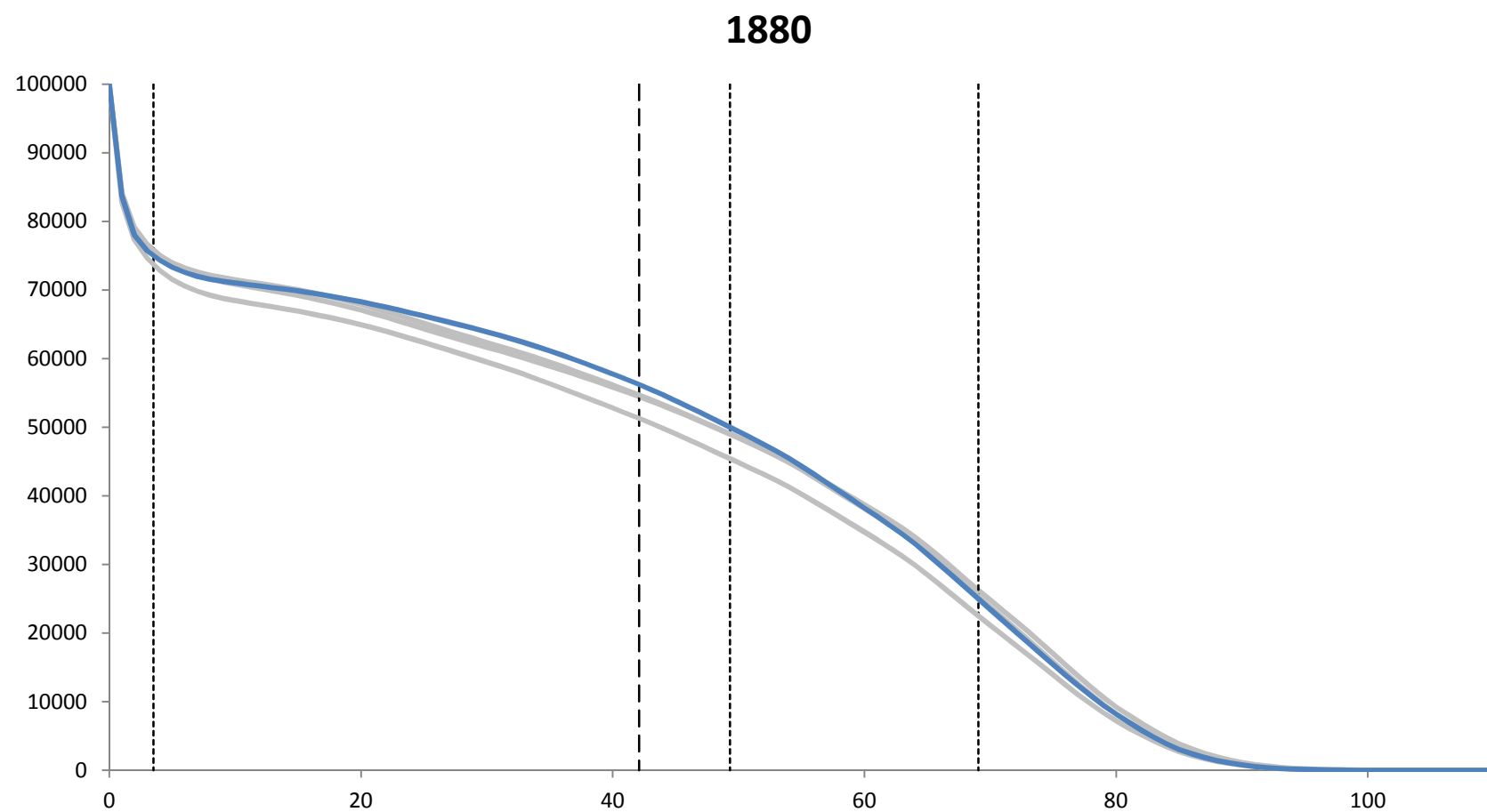
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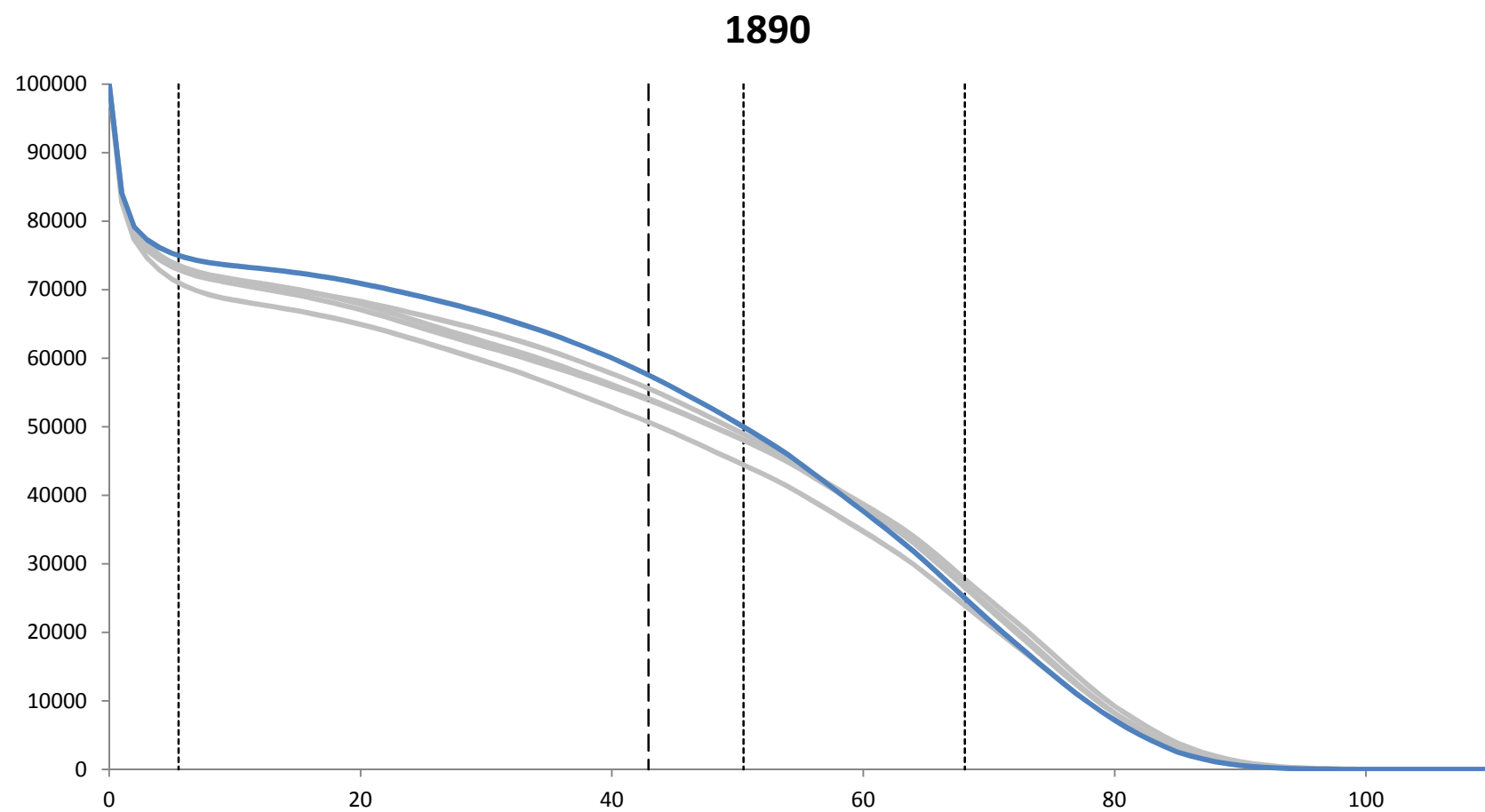
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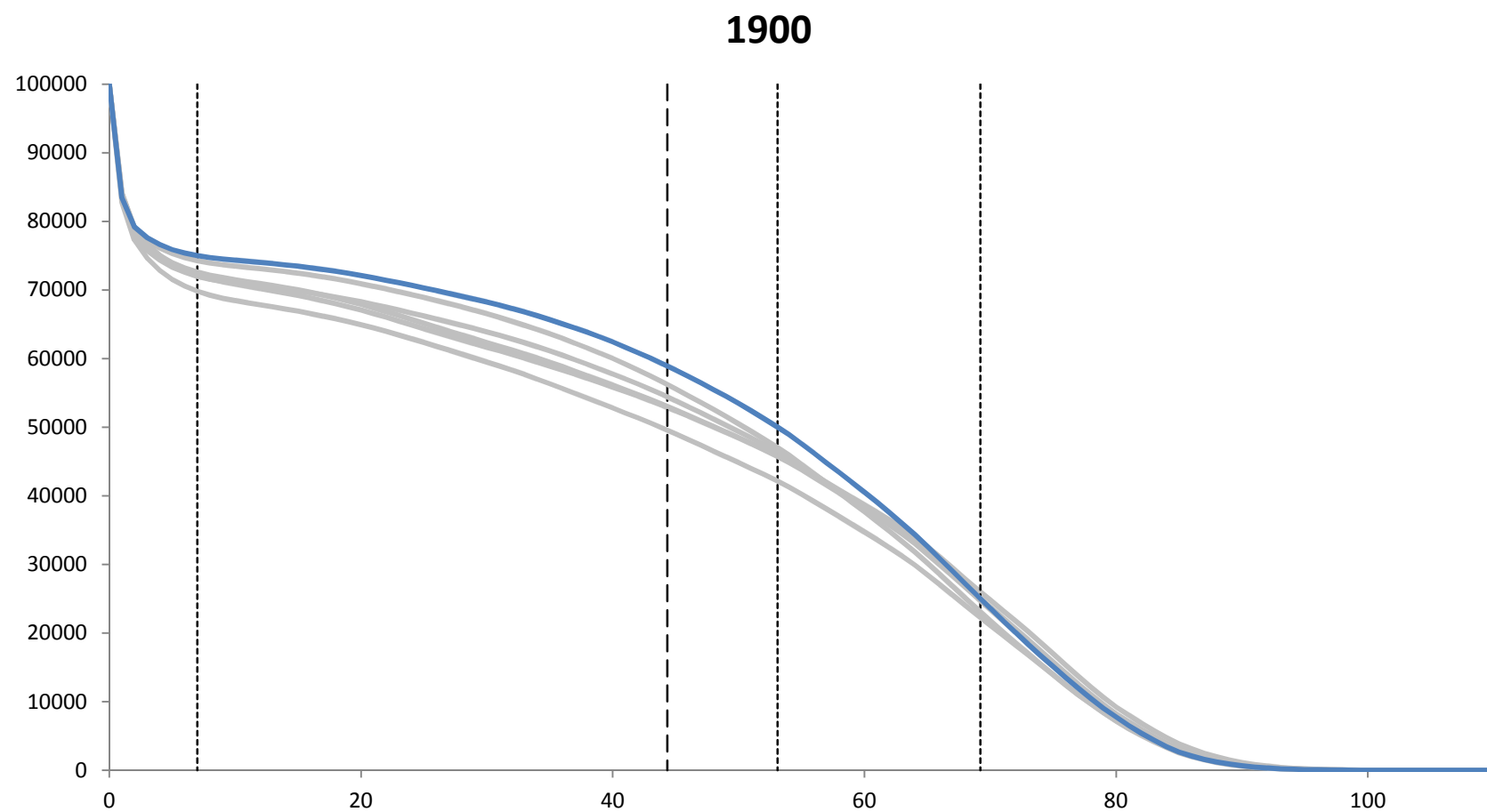
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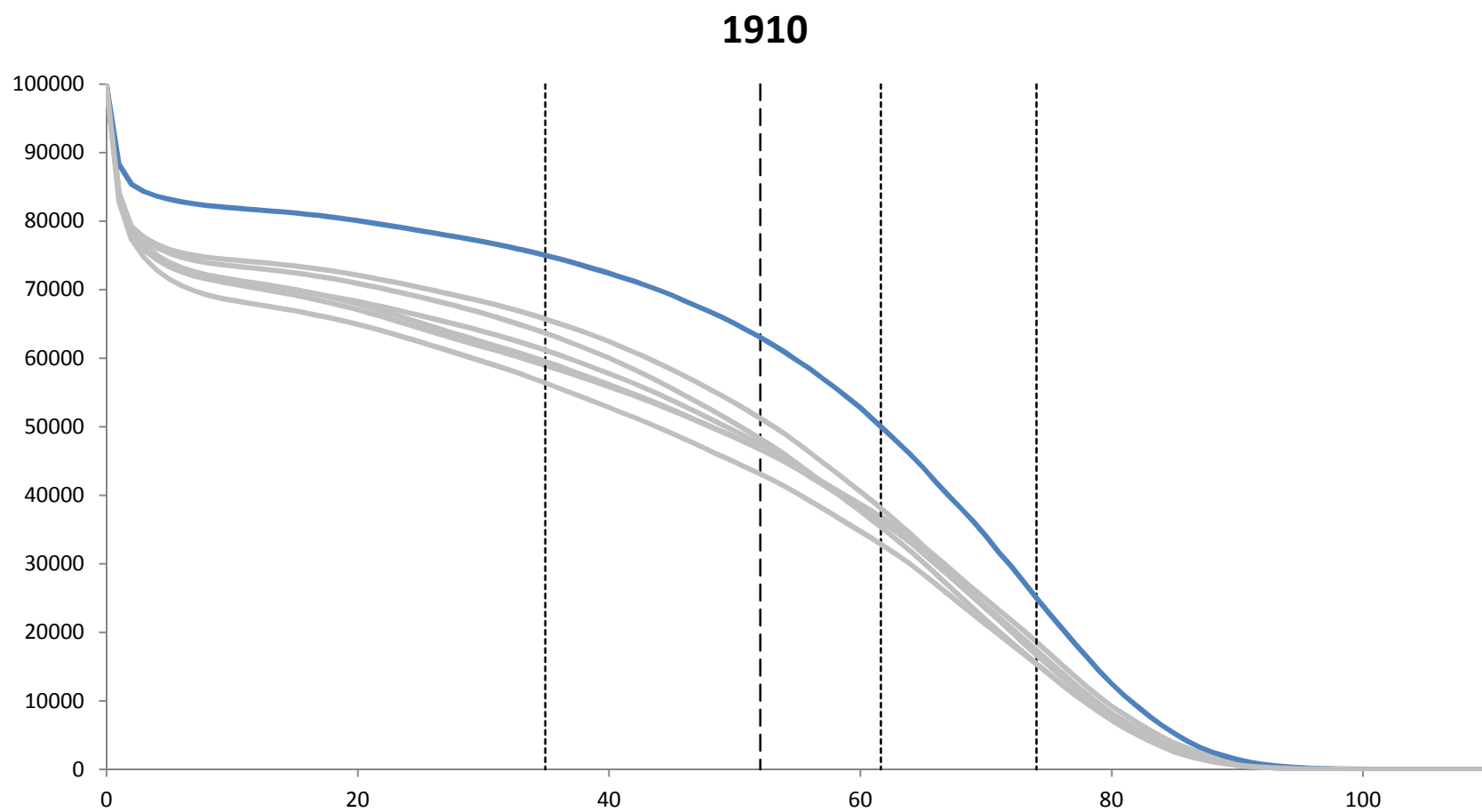
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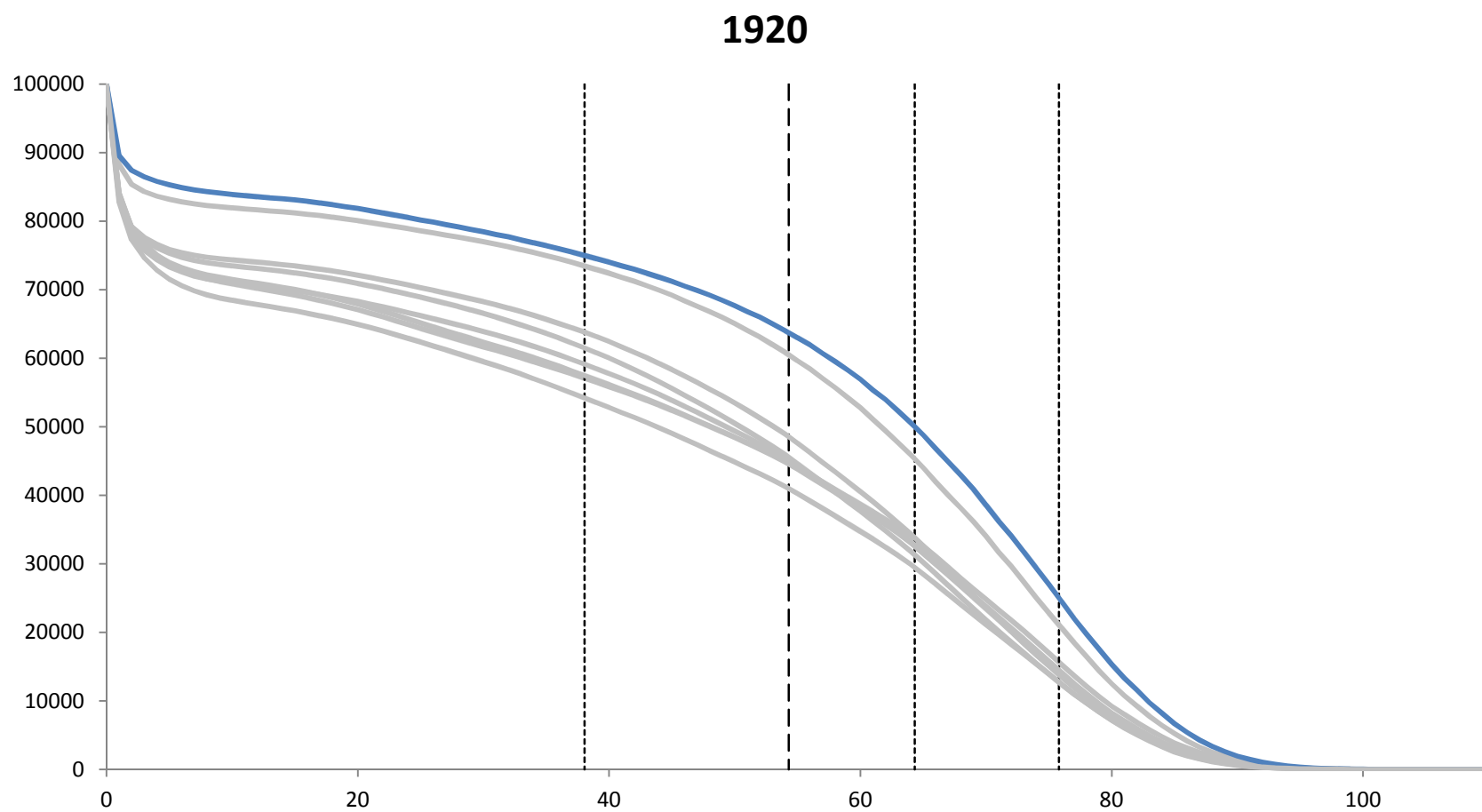
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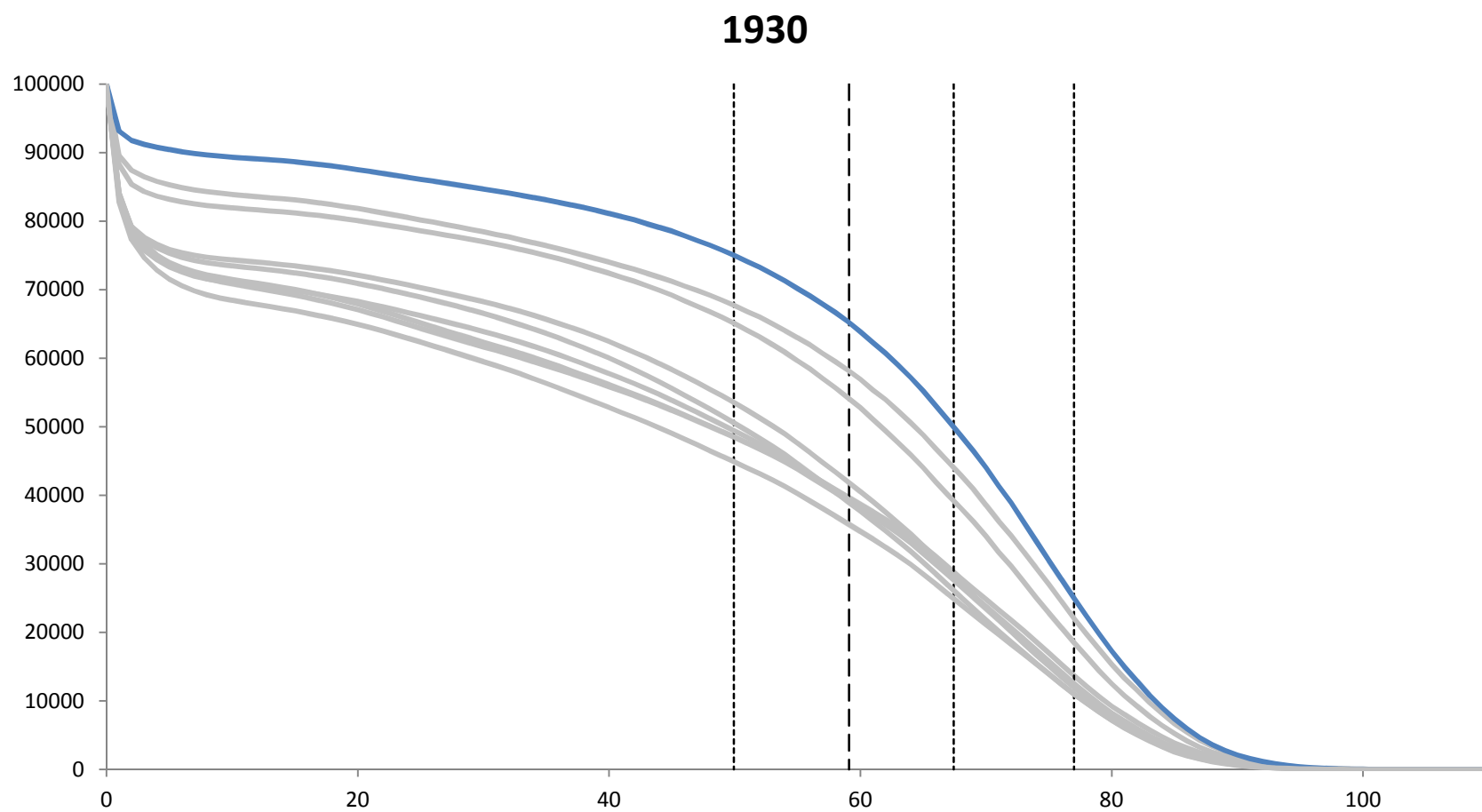
Source: Human Mortality Database

Male number of survivors, England and Wales 1850-2009



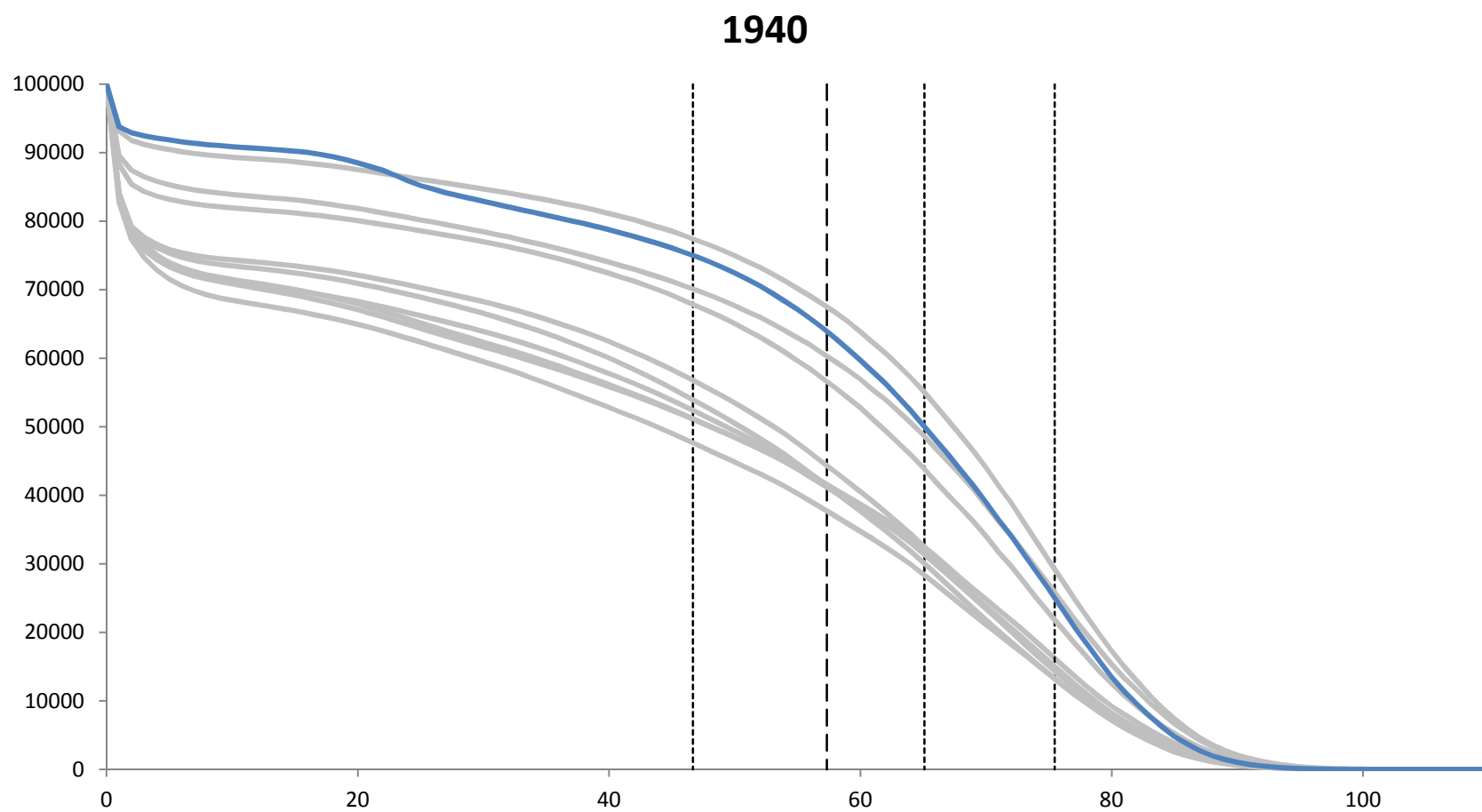
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Male number of survivors, England and Wales 1850-2009



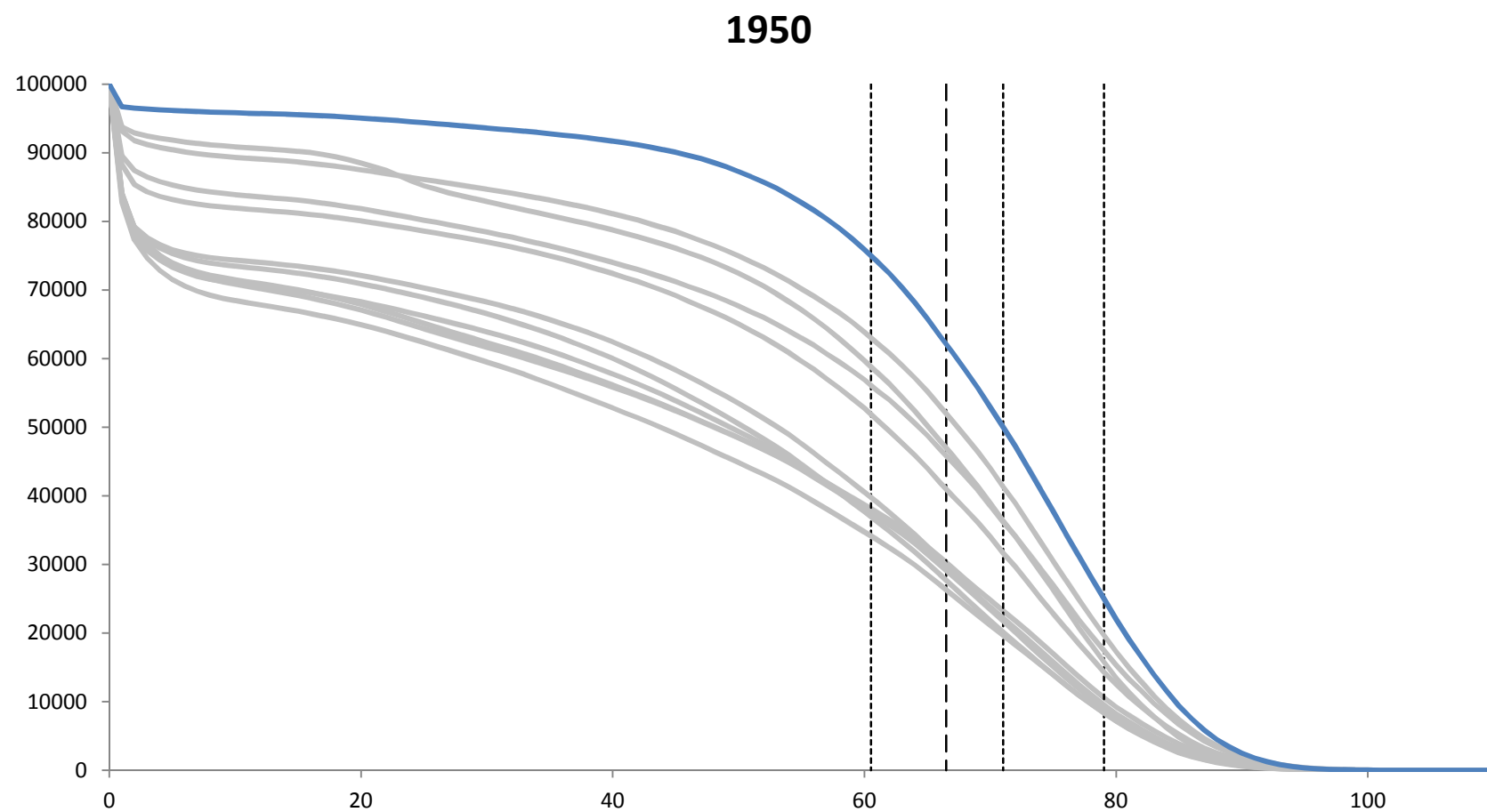
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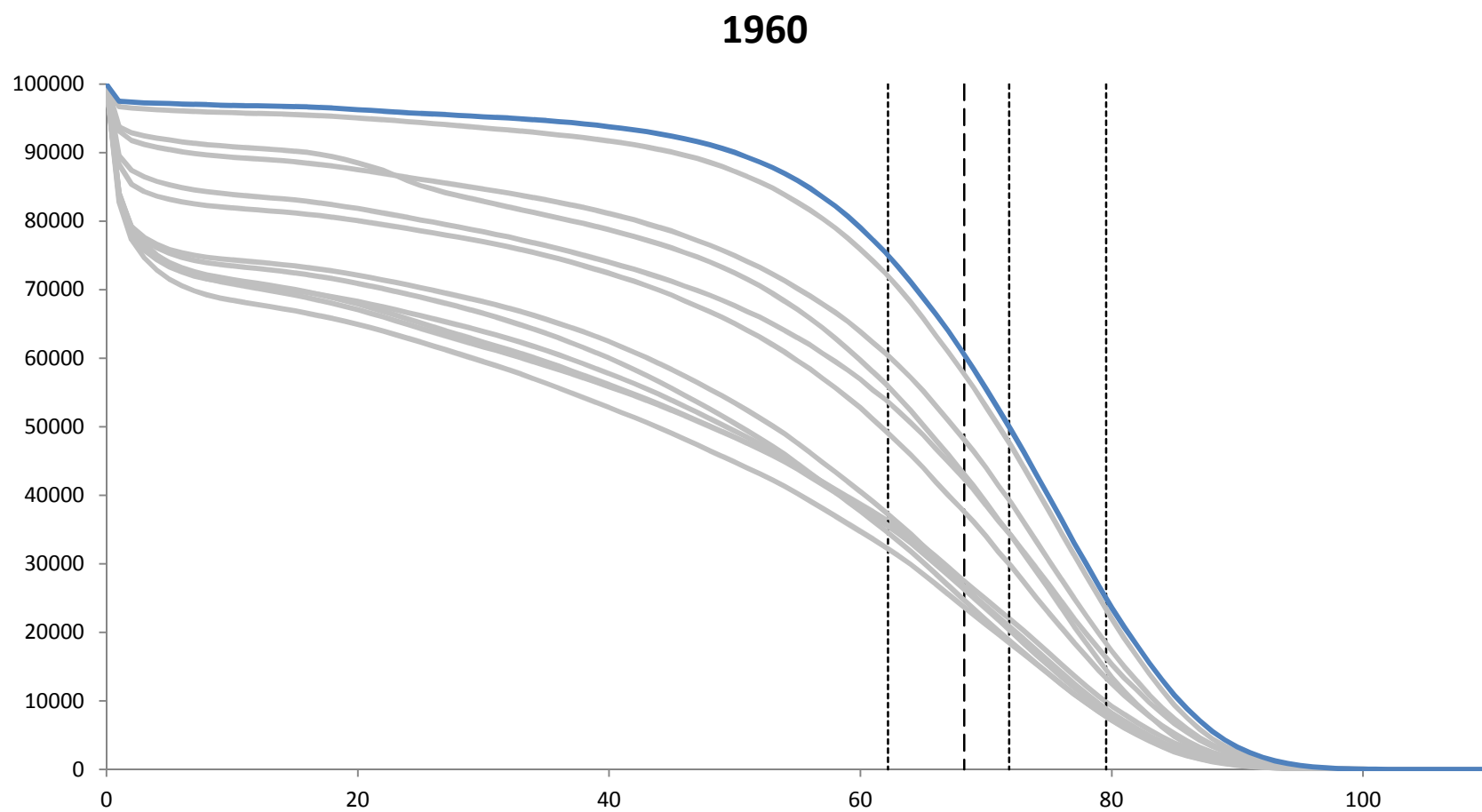
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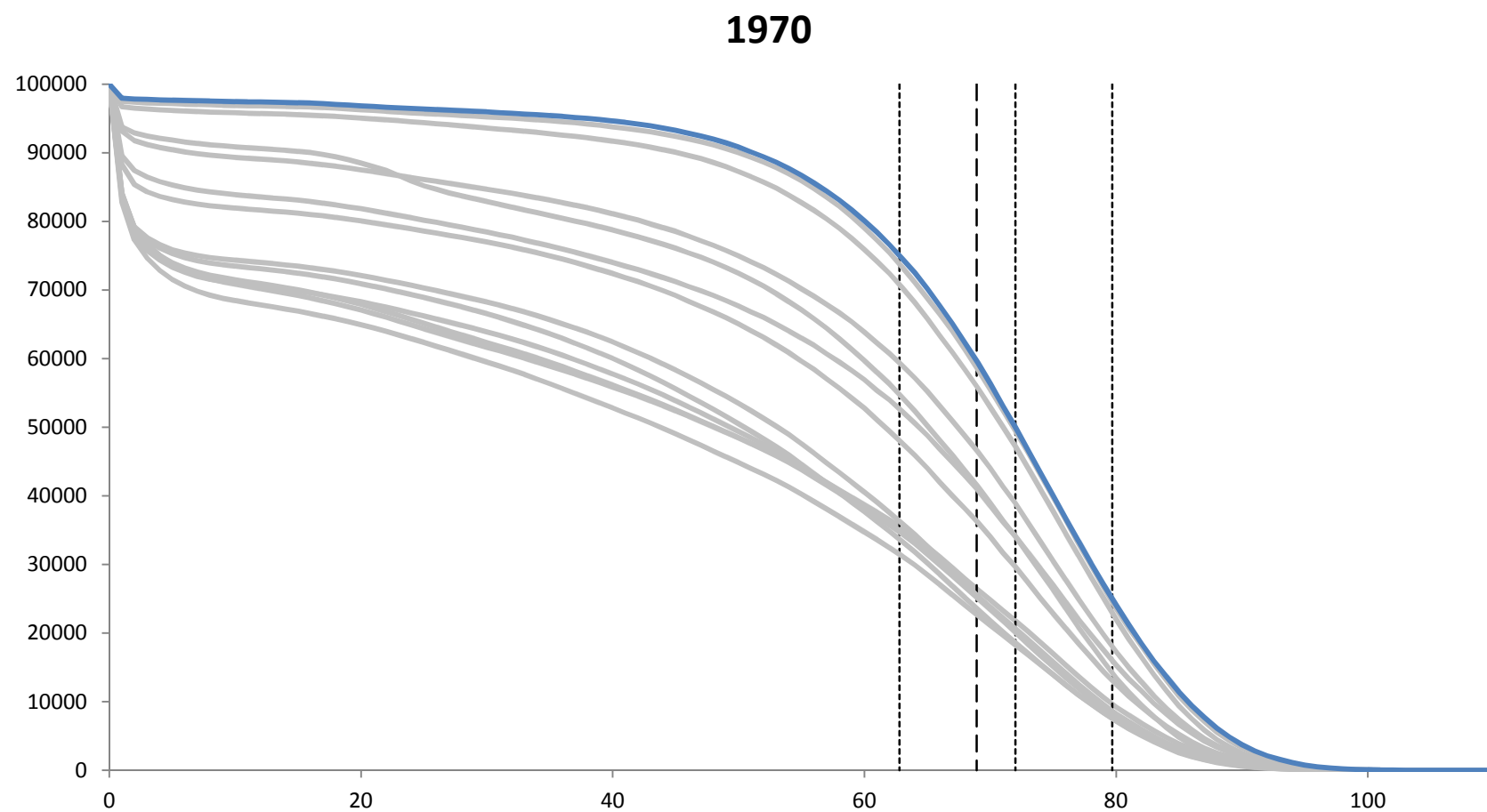
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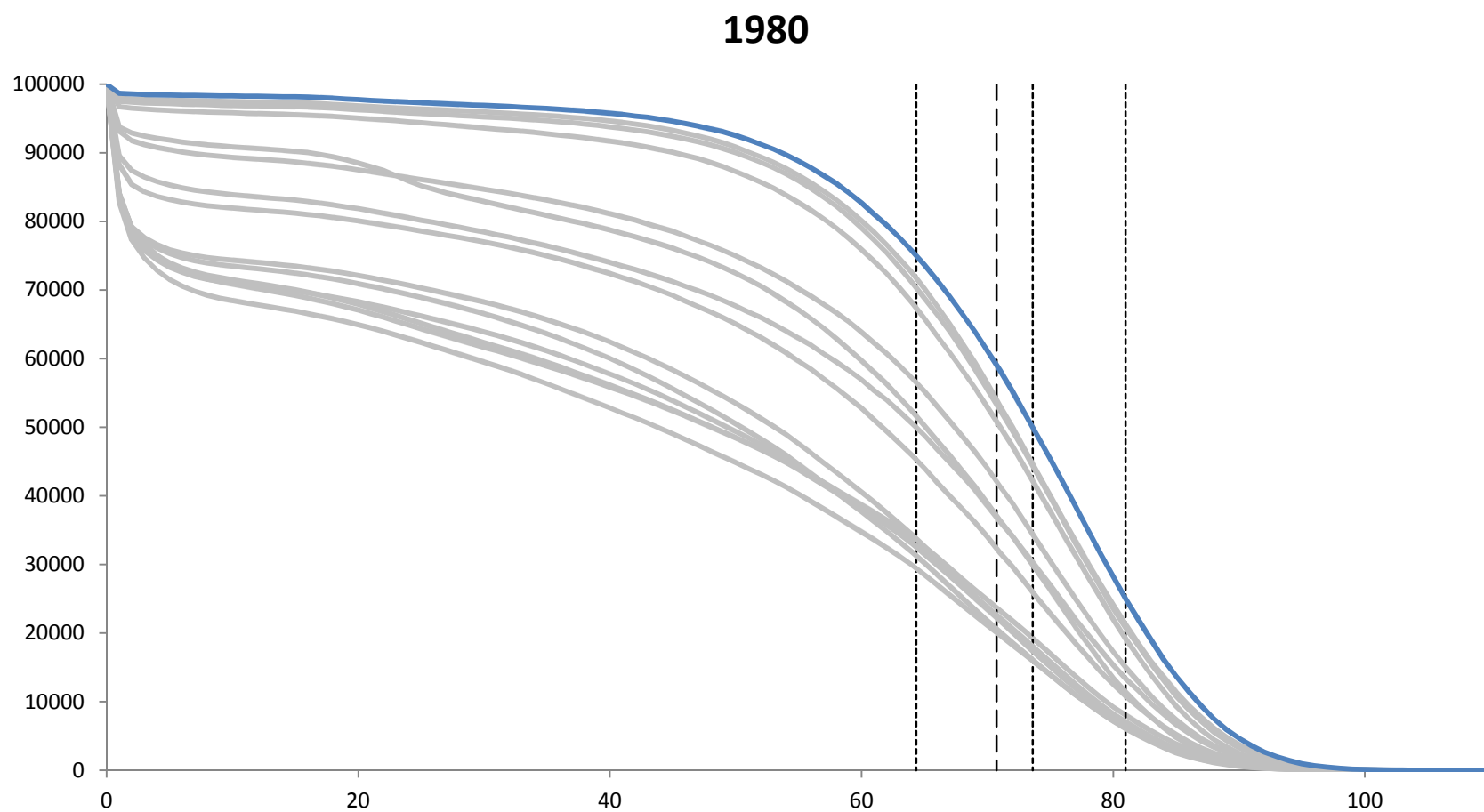
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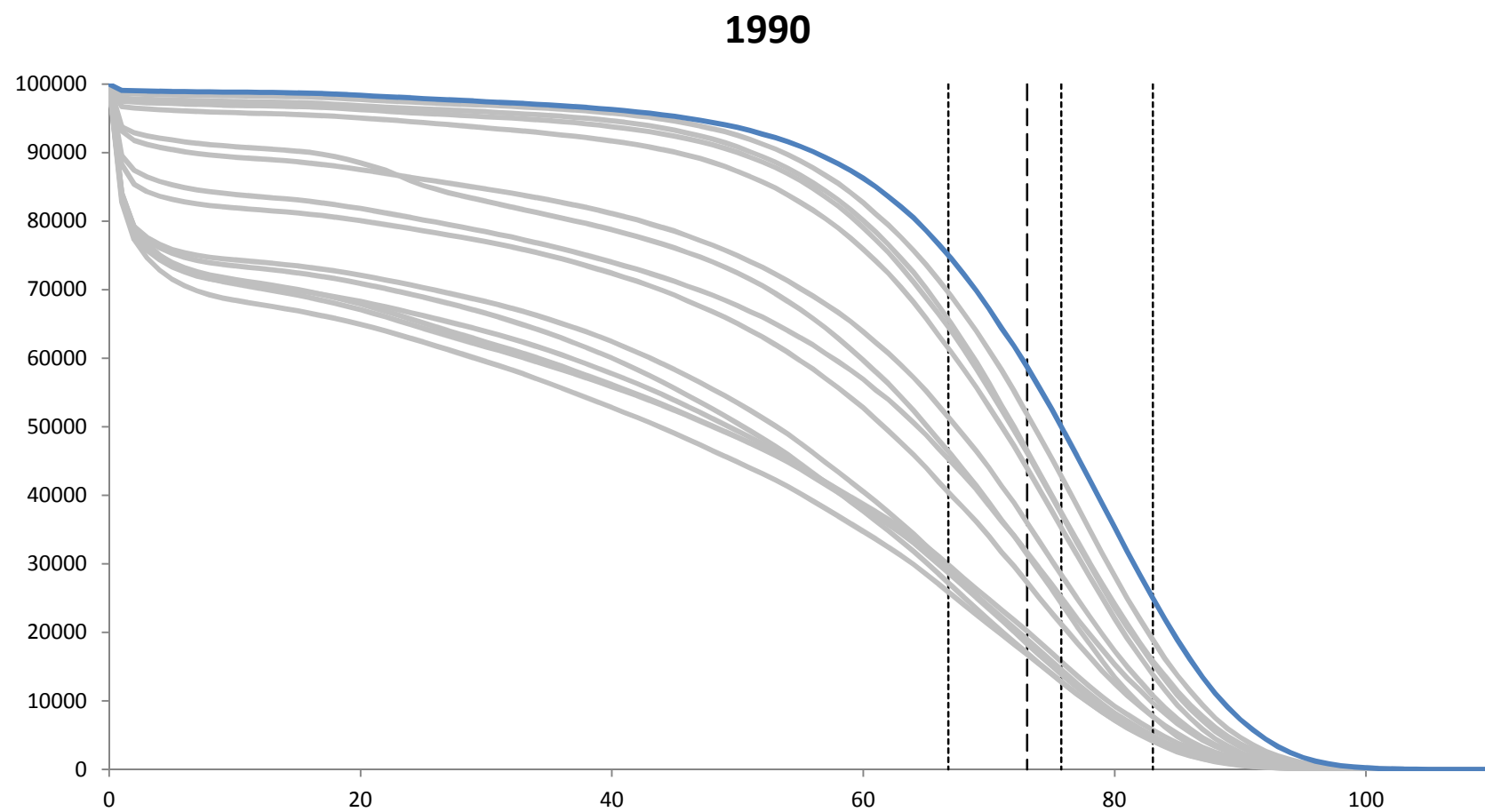
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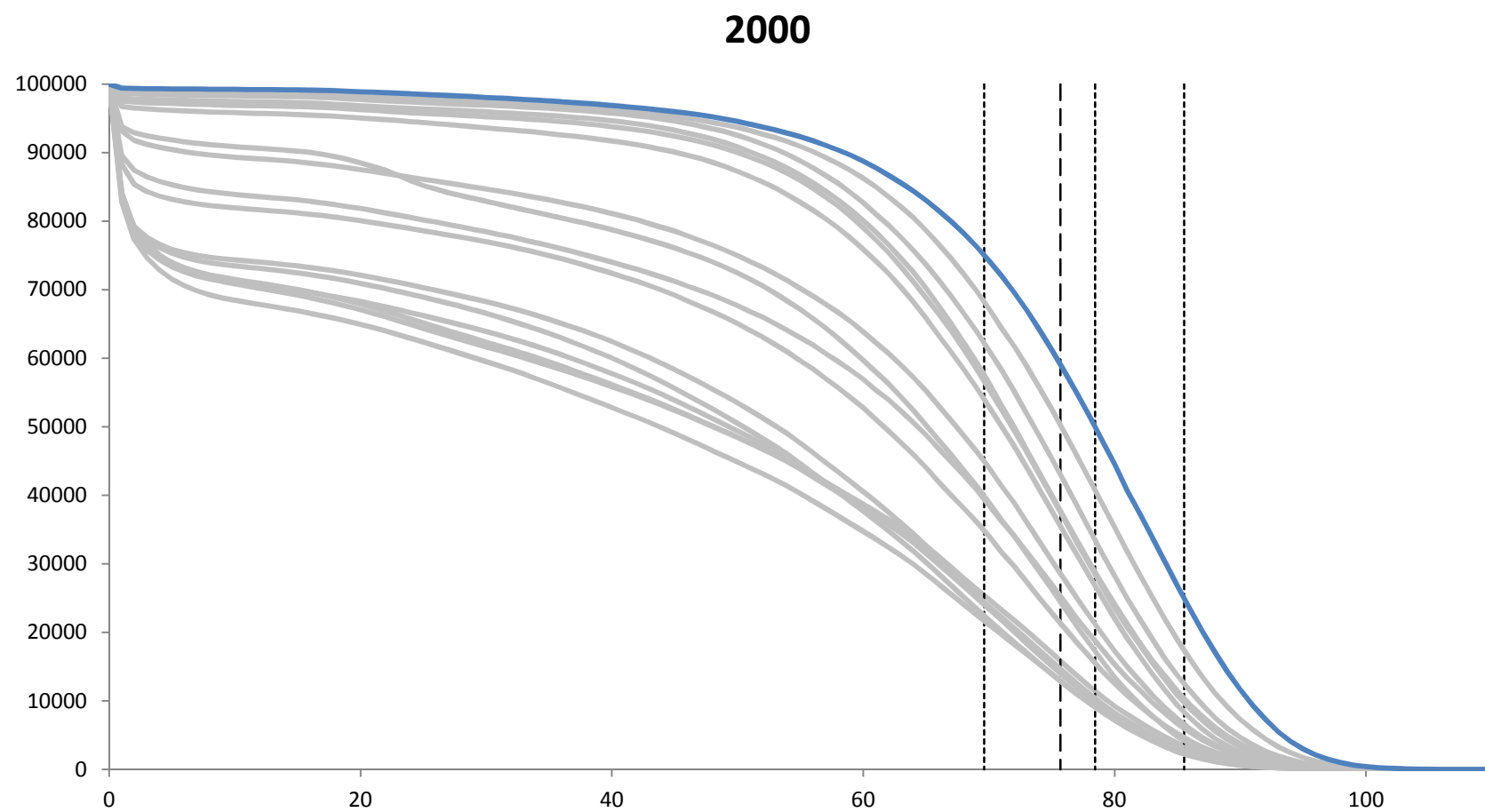
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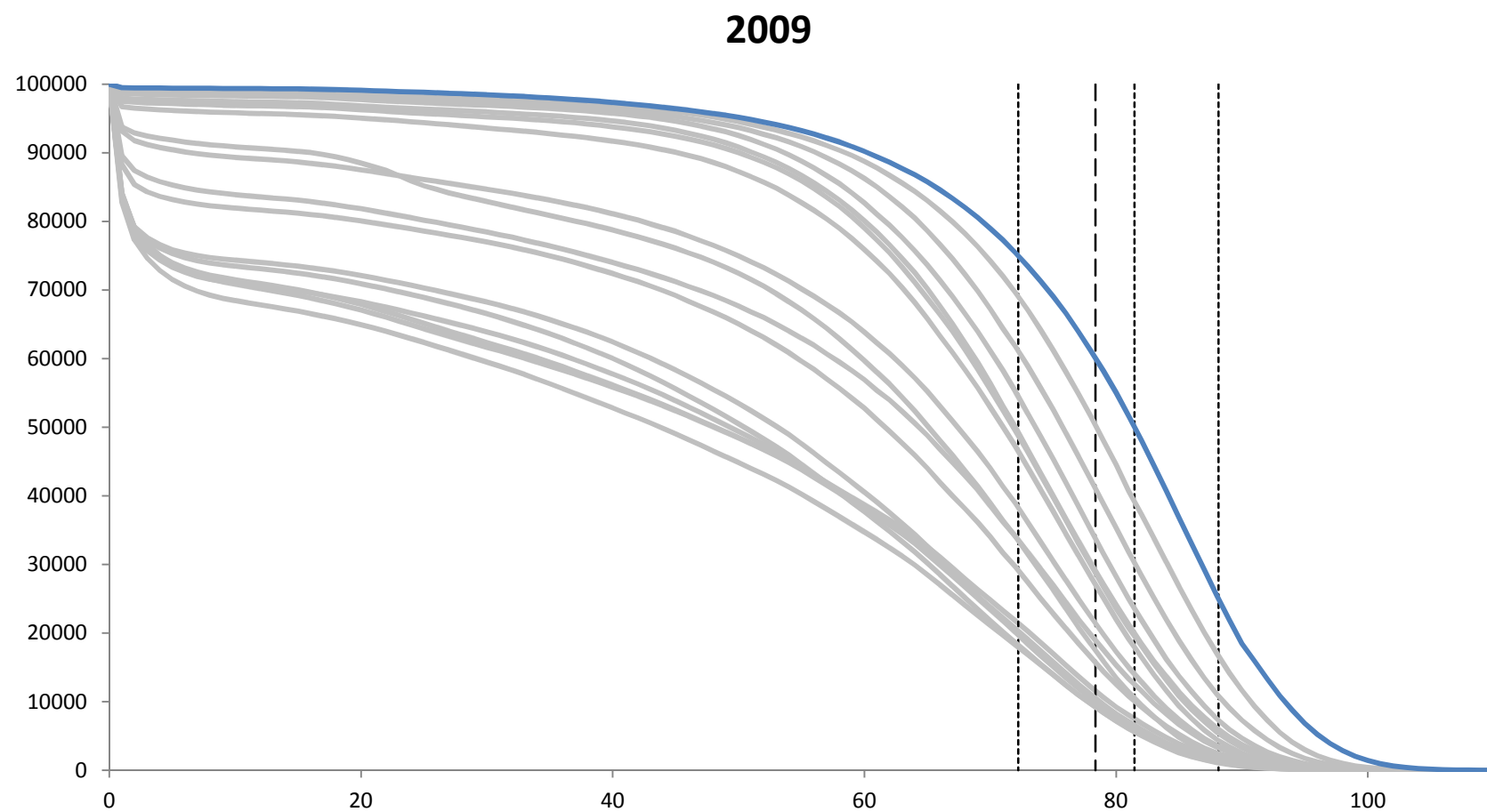
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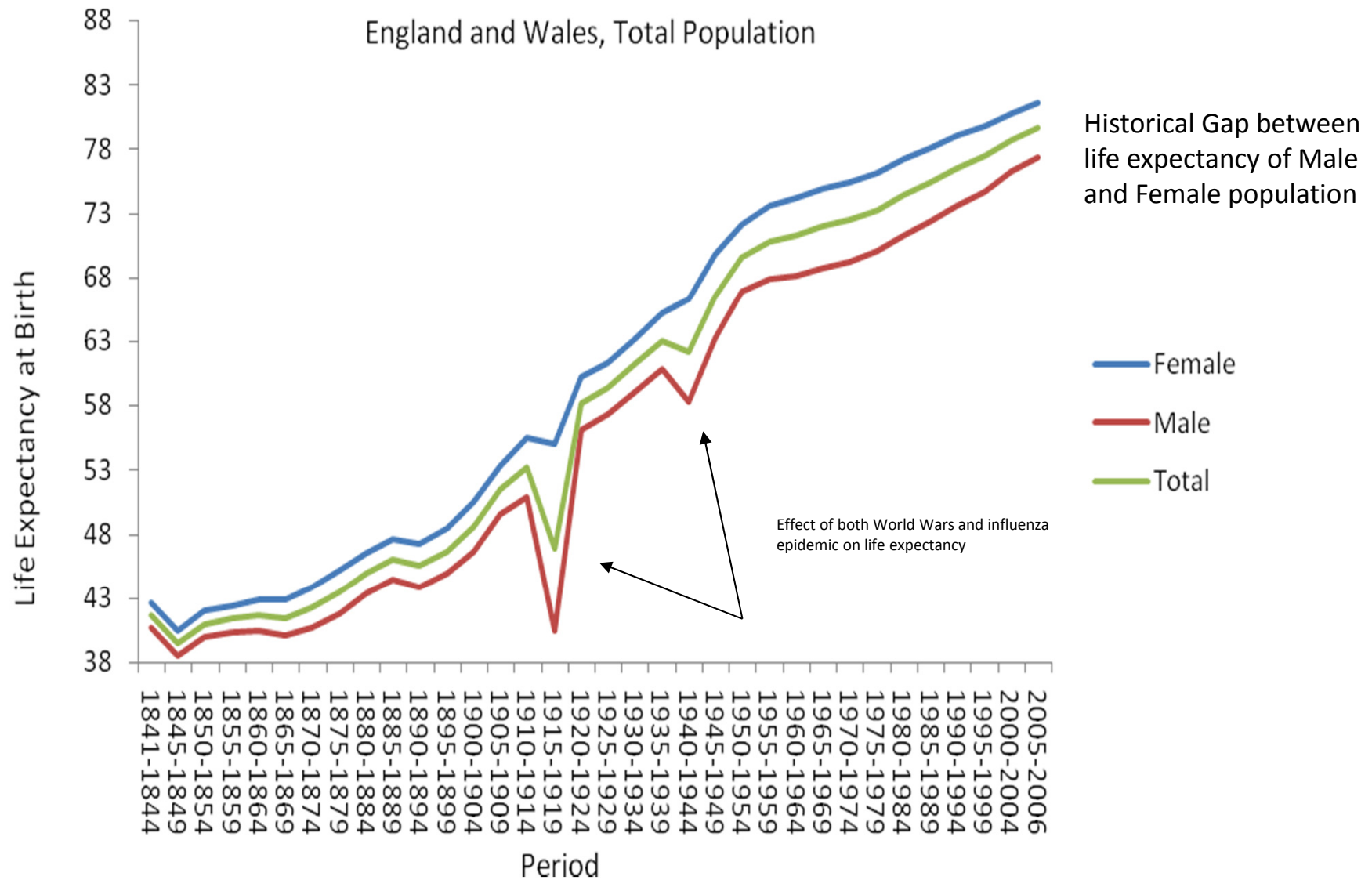
Source: Human Mortality Database

Male number of survivors, England and Wales 1850-2009



Source: Human Mortality Database

England and Wales Life Expectancy



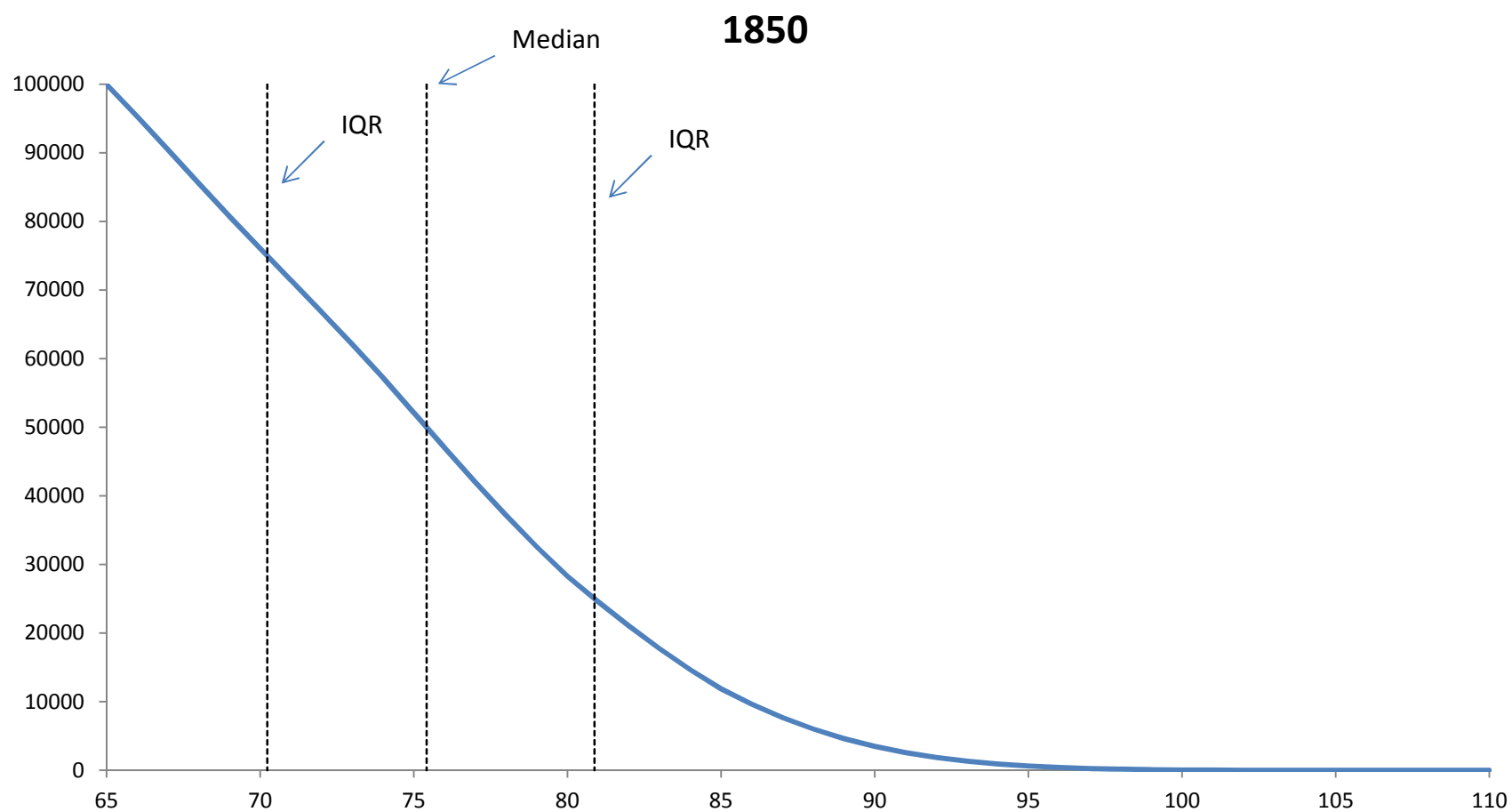
FOCUS ON OLDEST AGES

HISTORY OF LIFE EXPECTANCY

MALES AGED 65

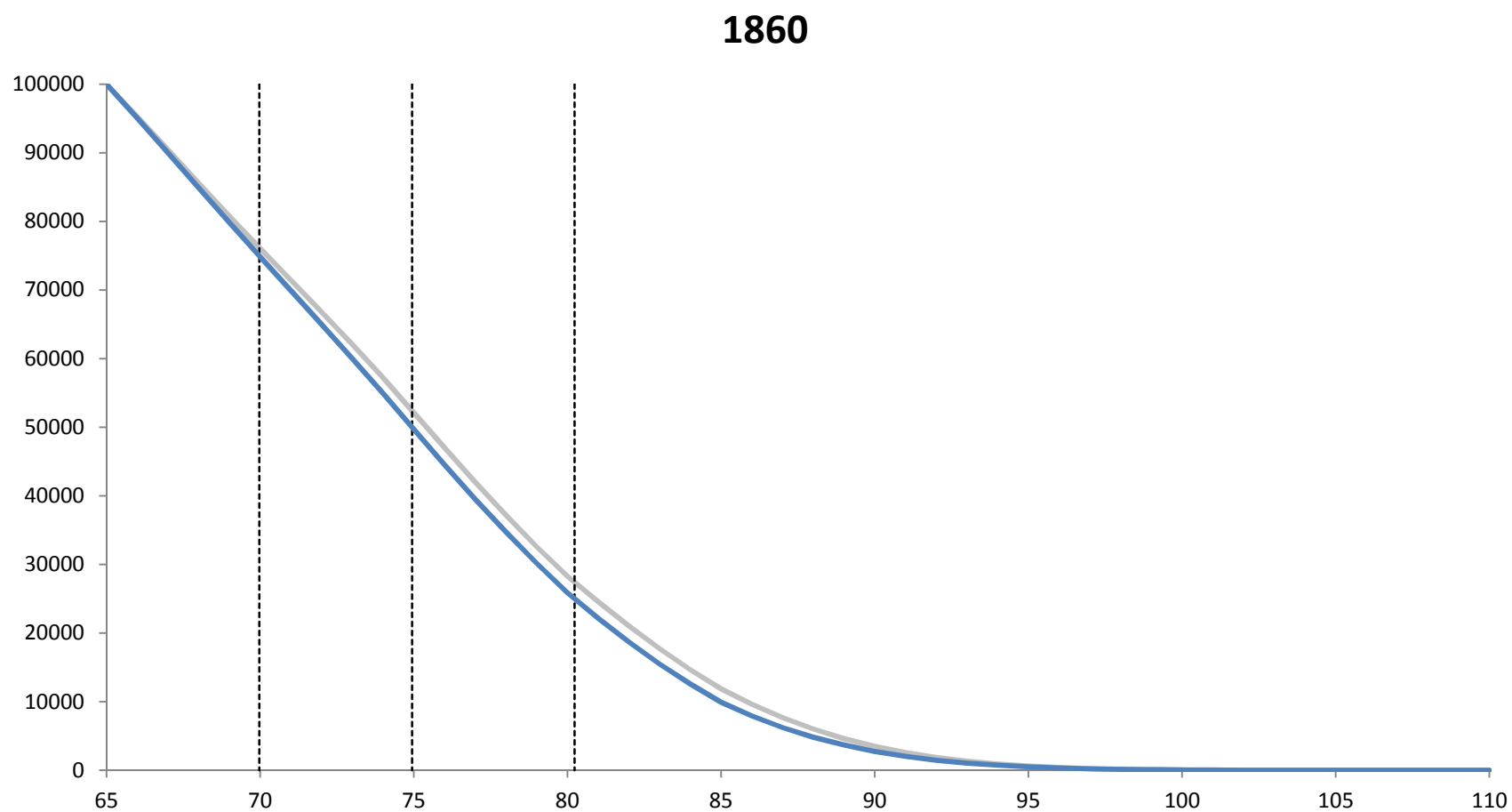
Mortality Table	Life Expectancy (years)
Ulpianus (230)	5.3
Breslau-Halley (1693)	9.6
Karlsruhe (1864)	10.3
England and Wales (2000)	15.4

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009



Source: Human Mortality Database

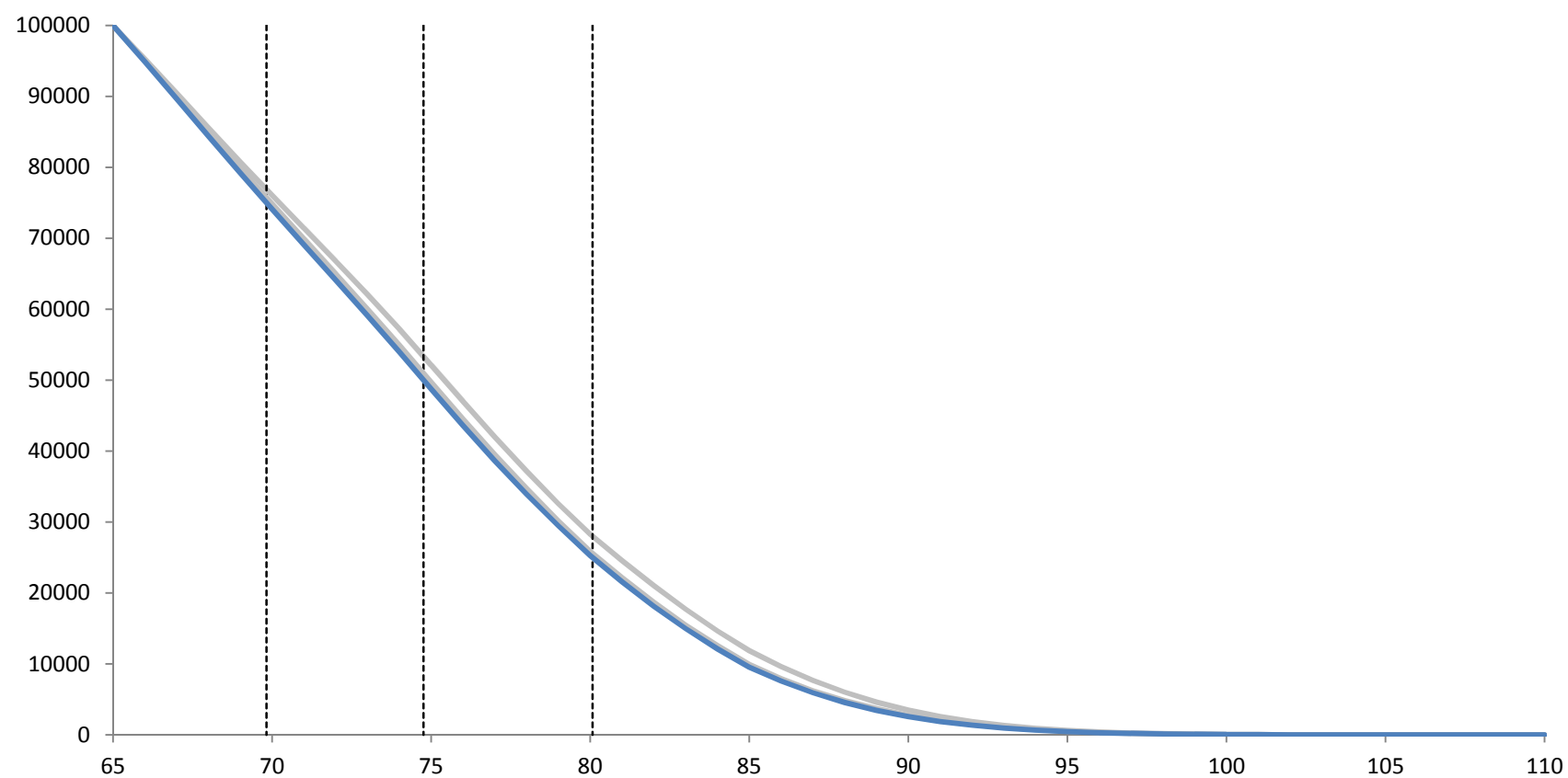
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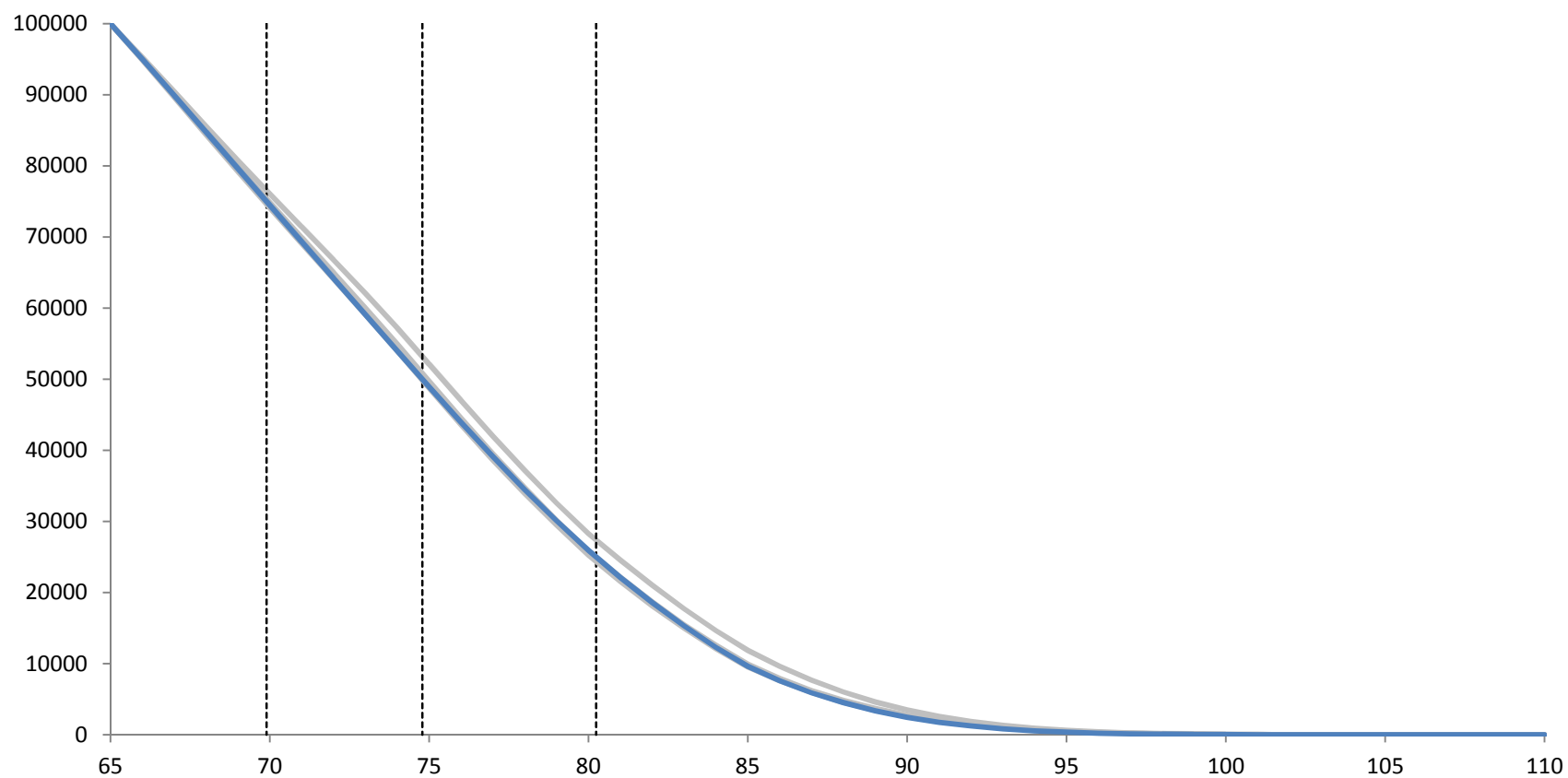
1870



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

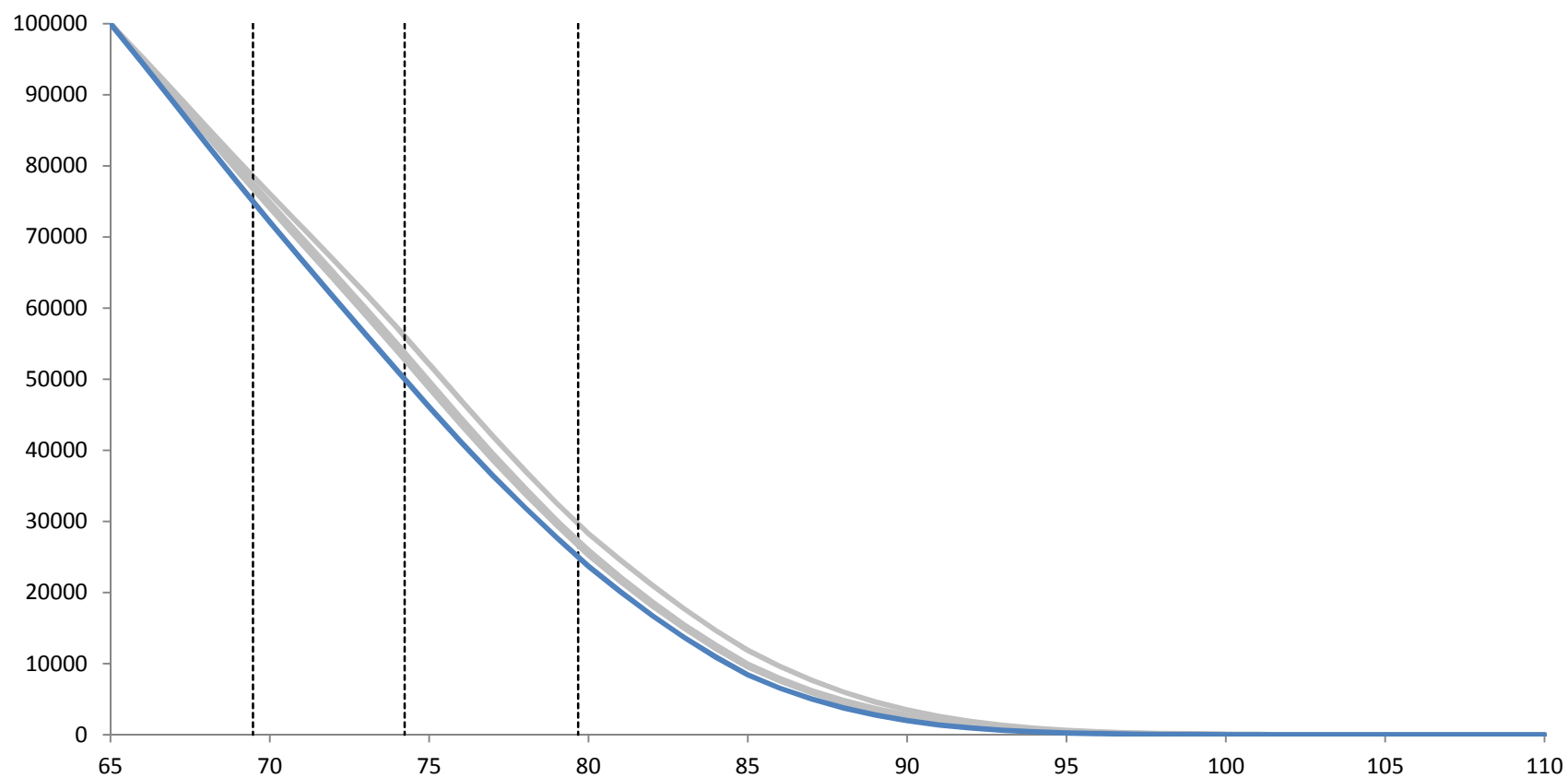
1880



Source: Human Mortality Database

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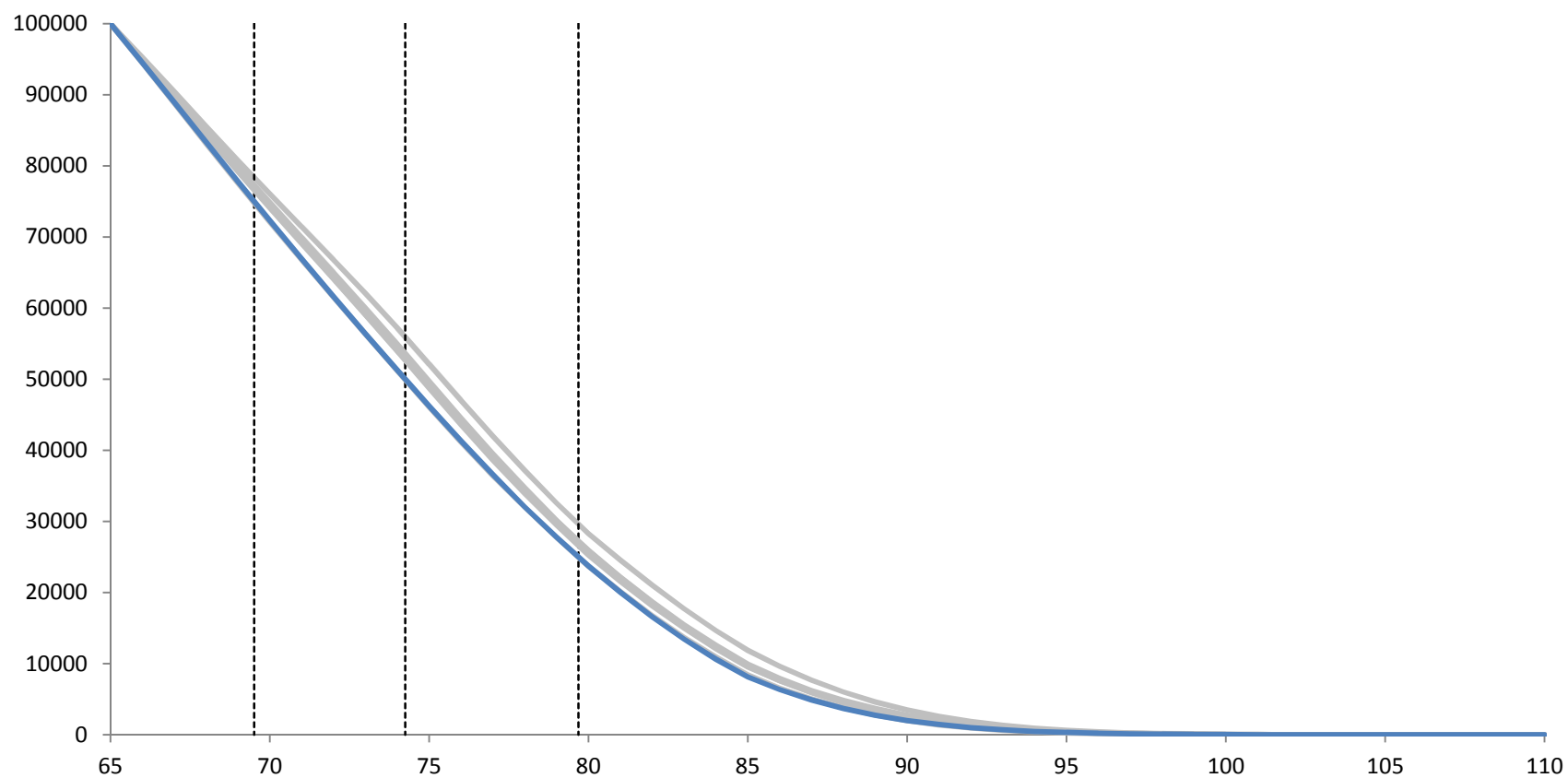
1890



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

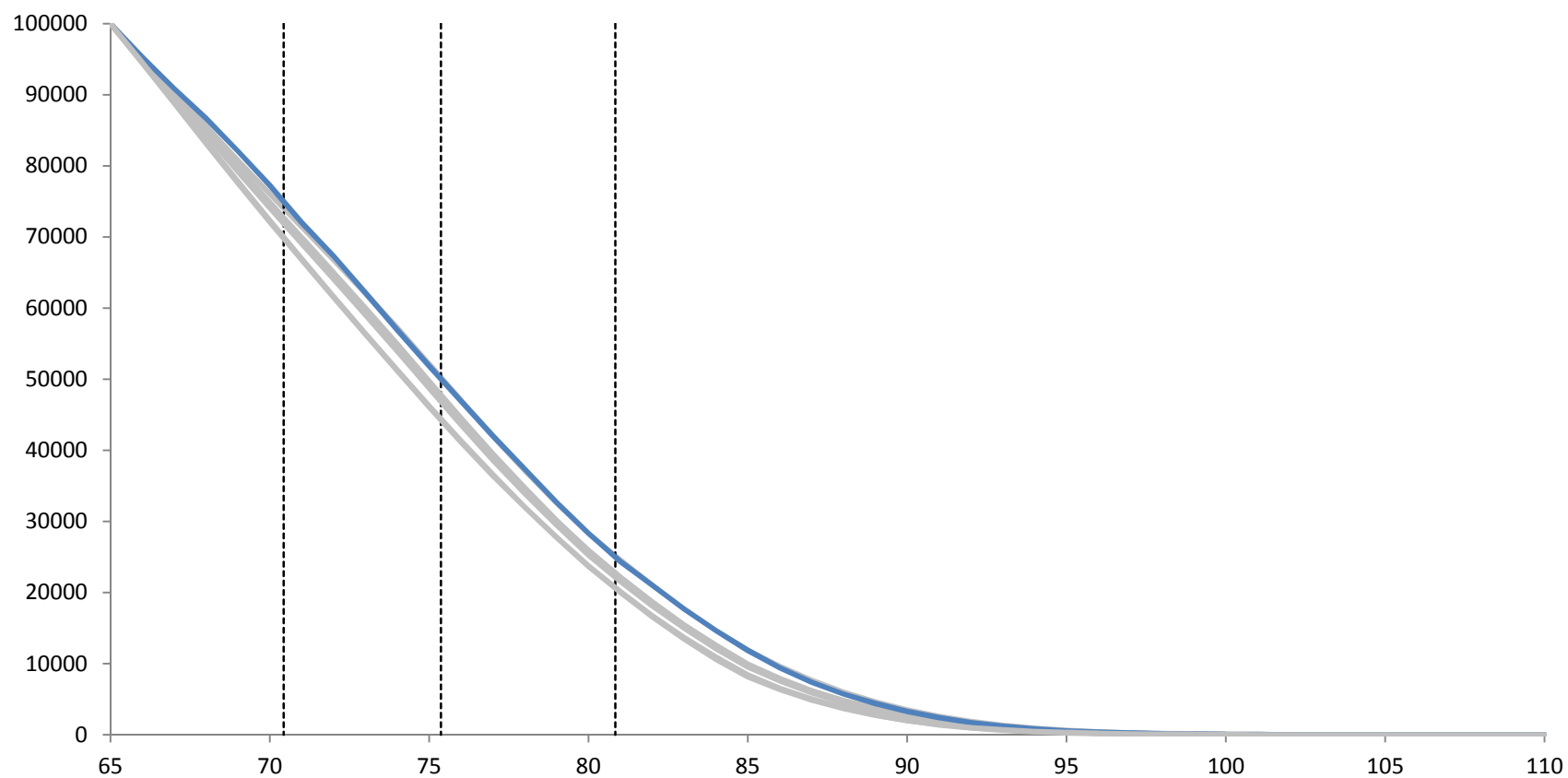
1900



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

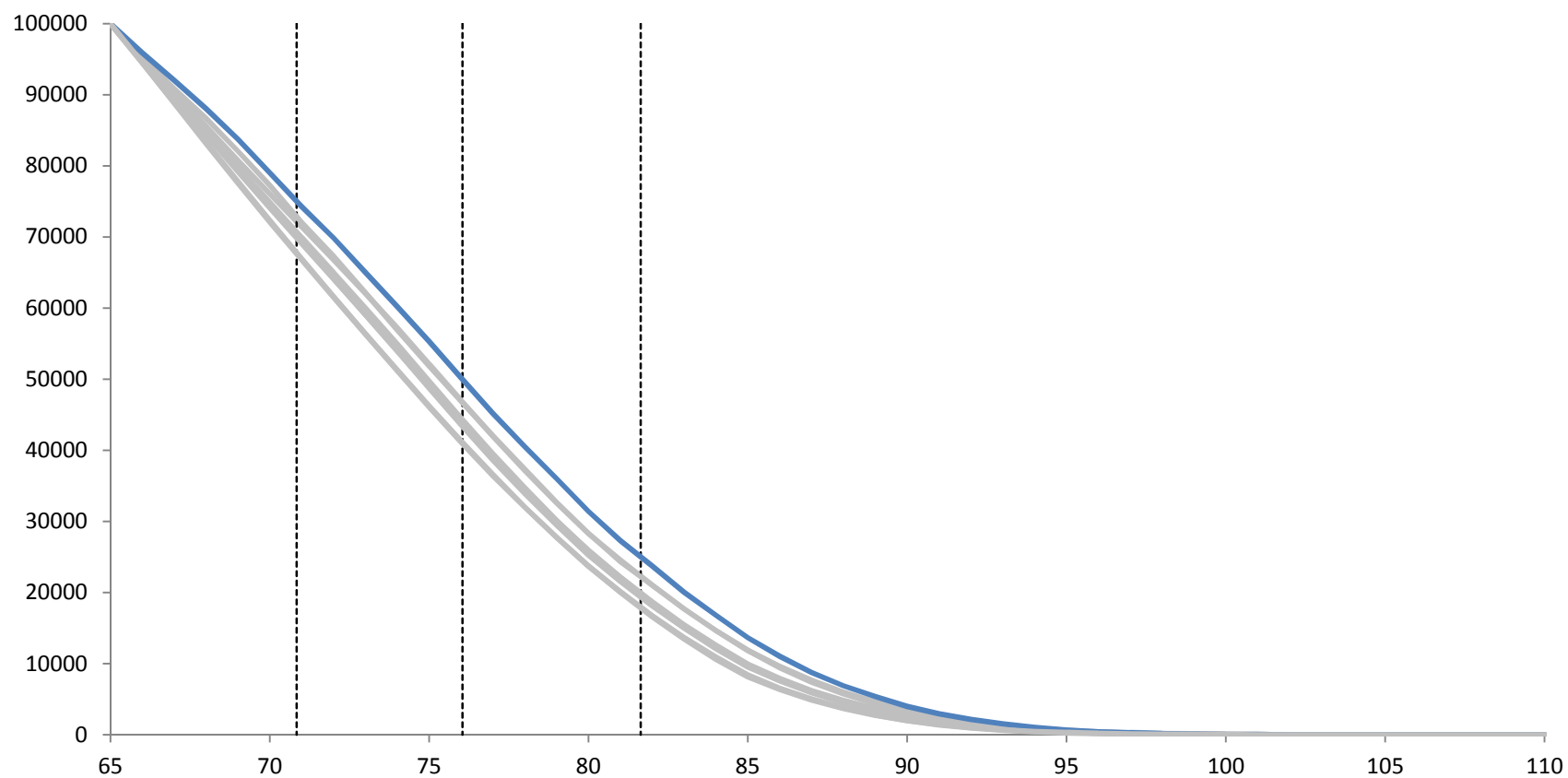
1910



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

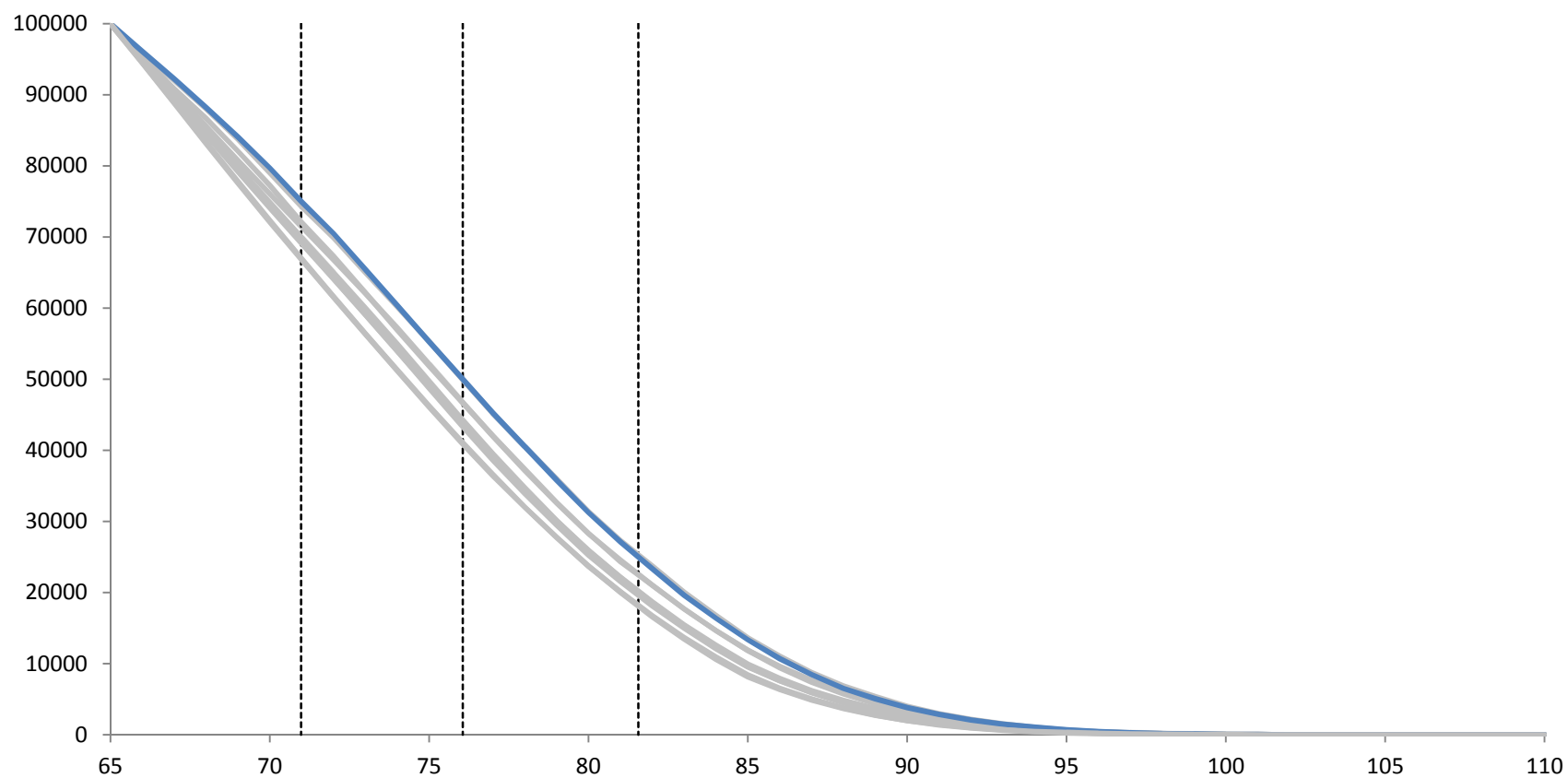
1920



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

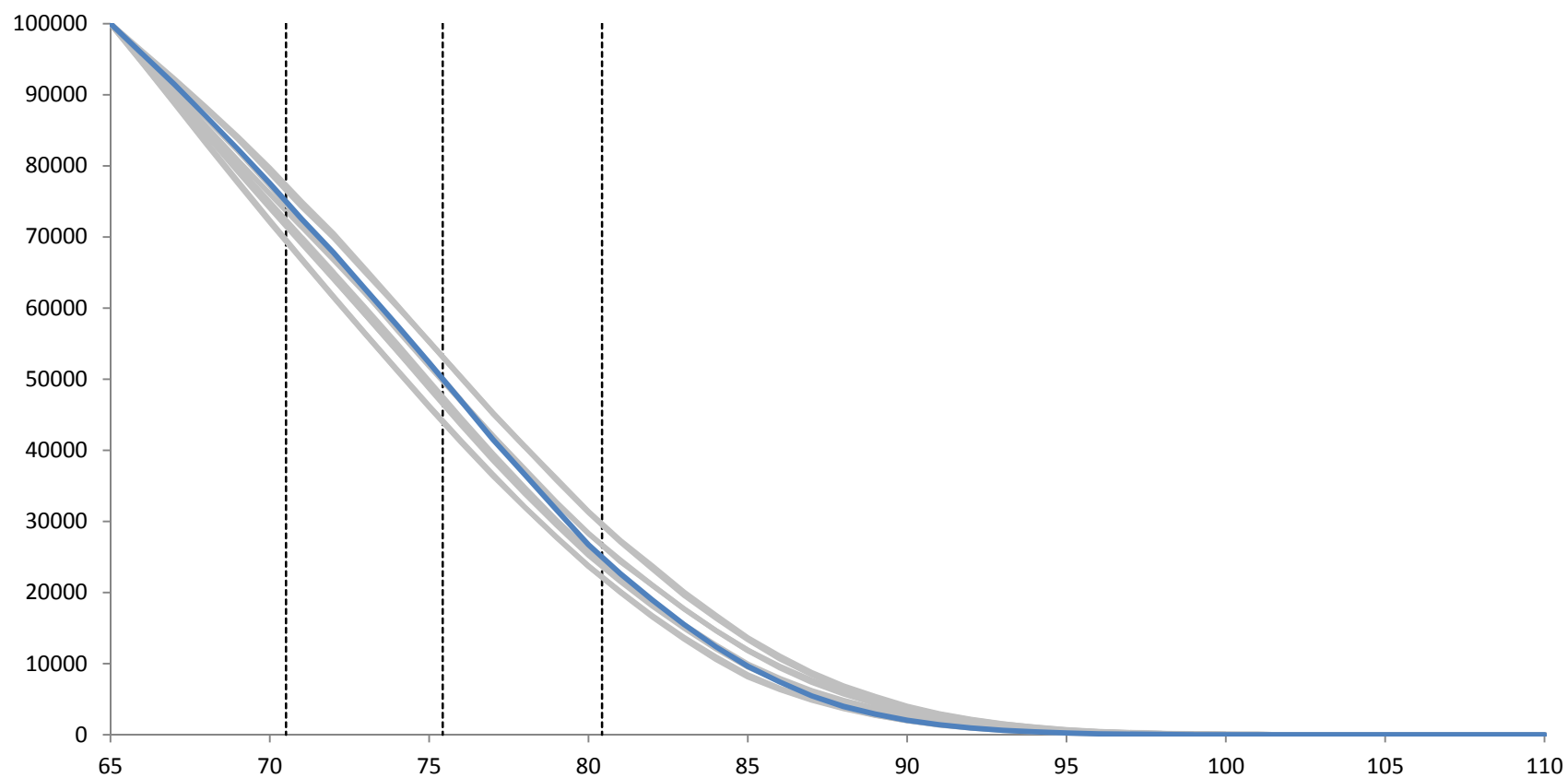
1930



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

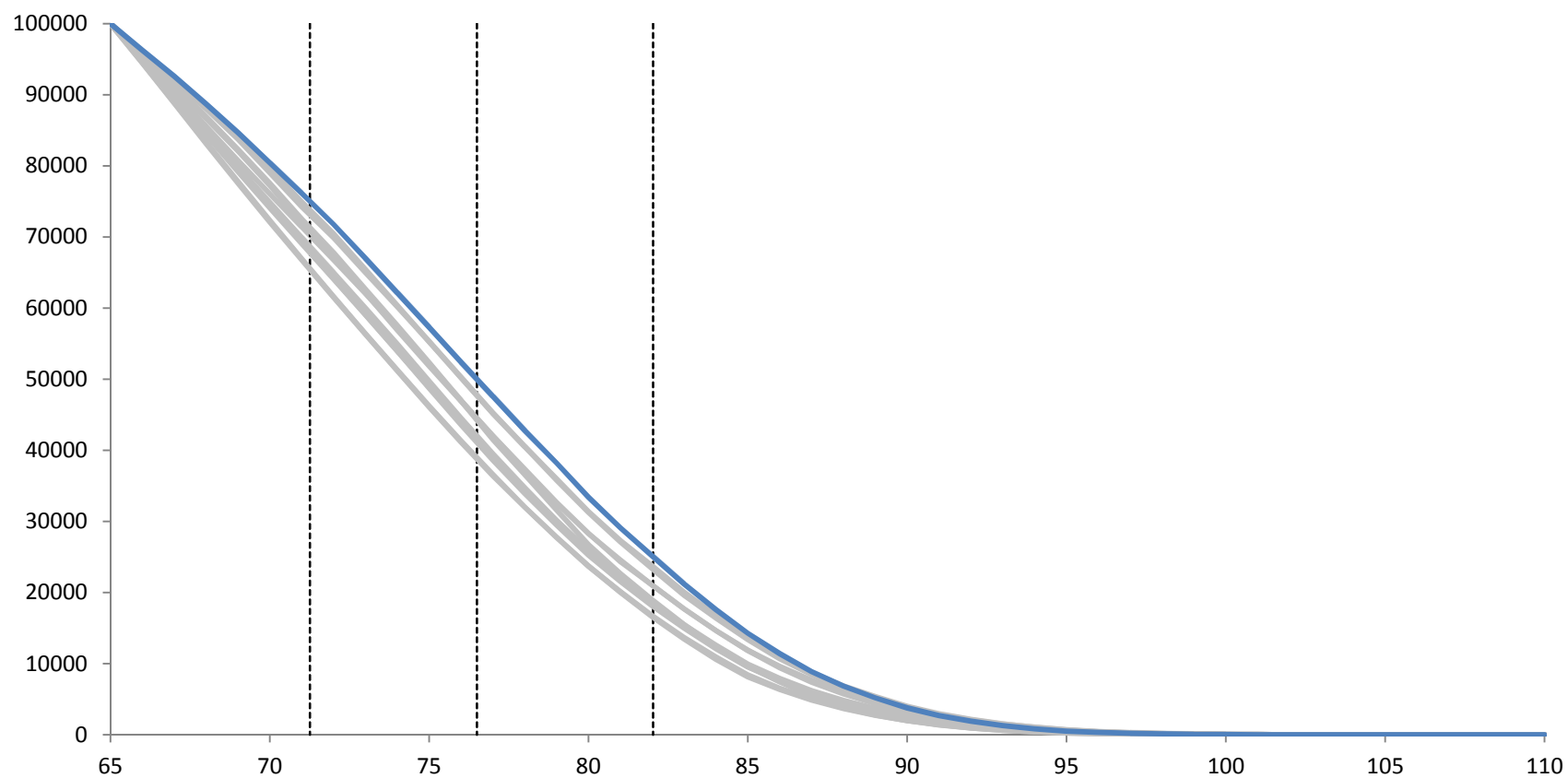
1940



Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

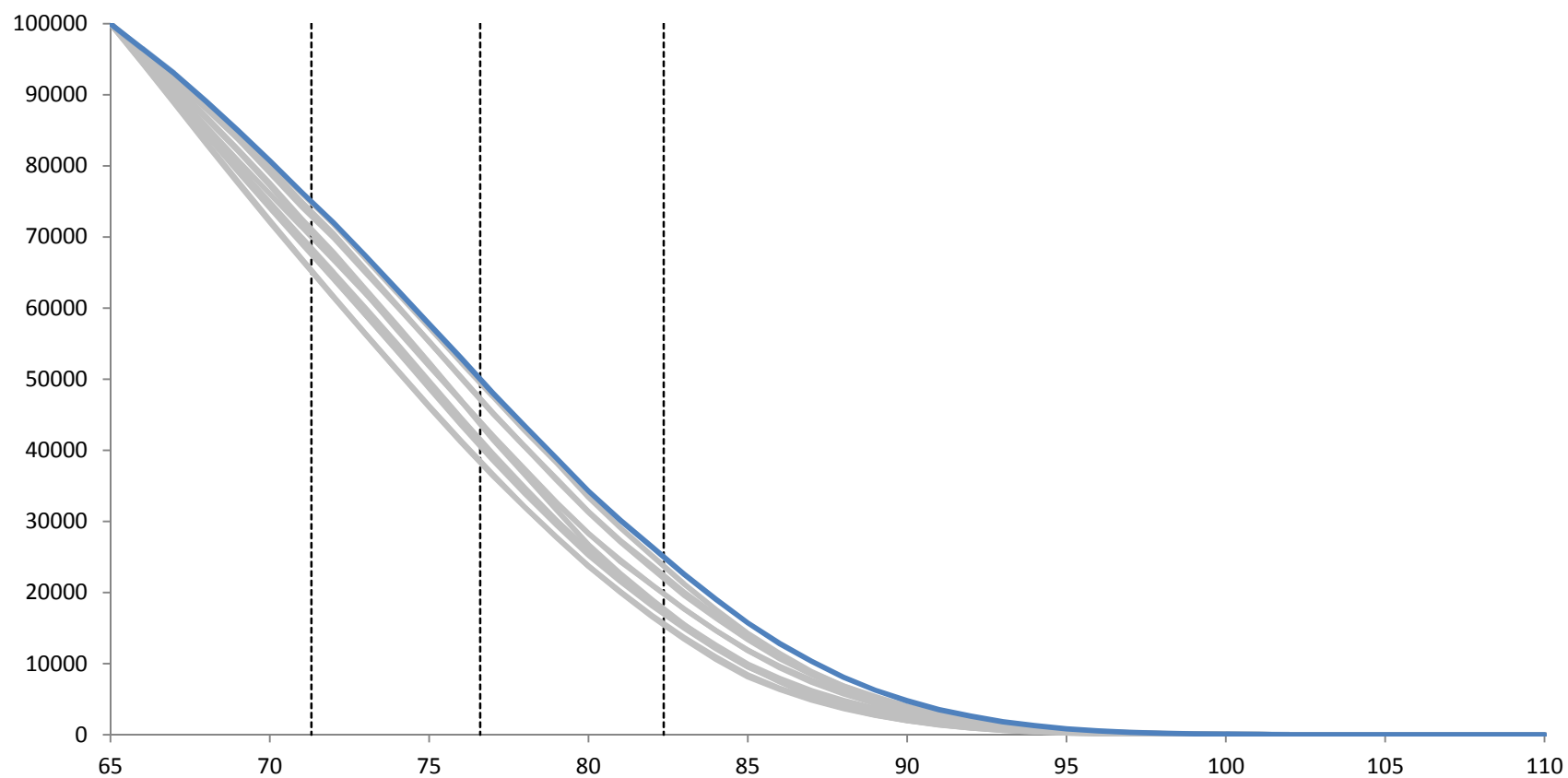
1950



Source: Human Mortality Database

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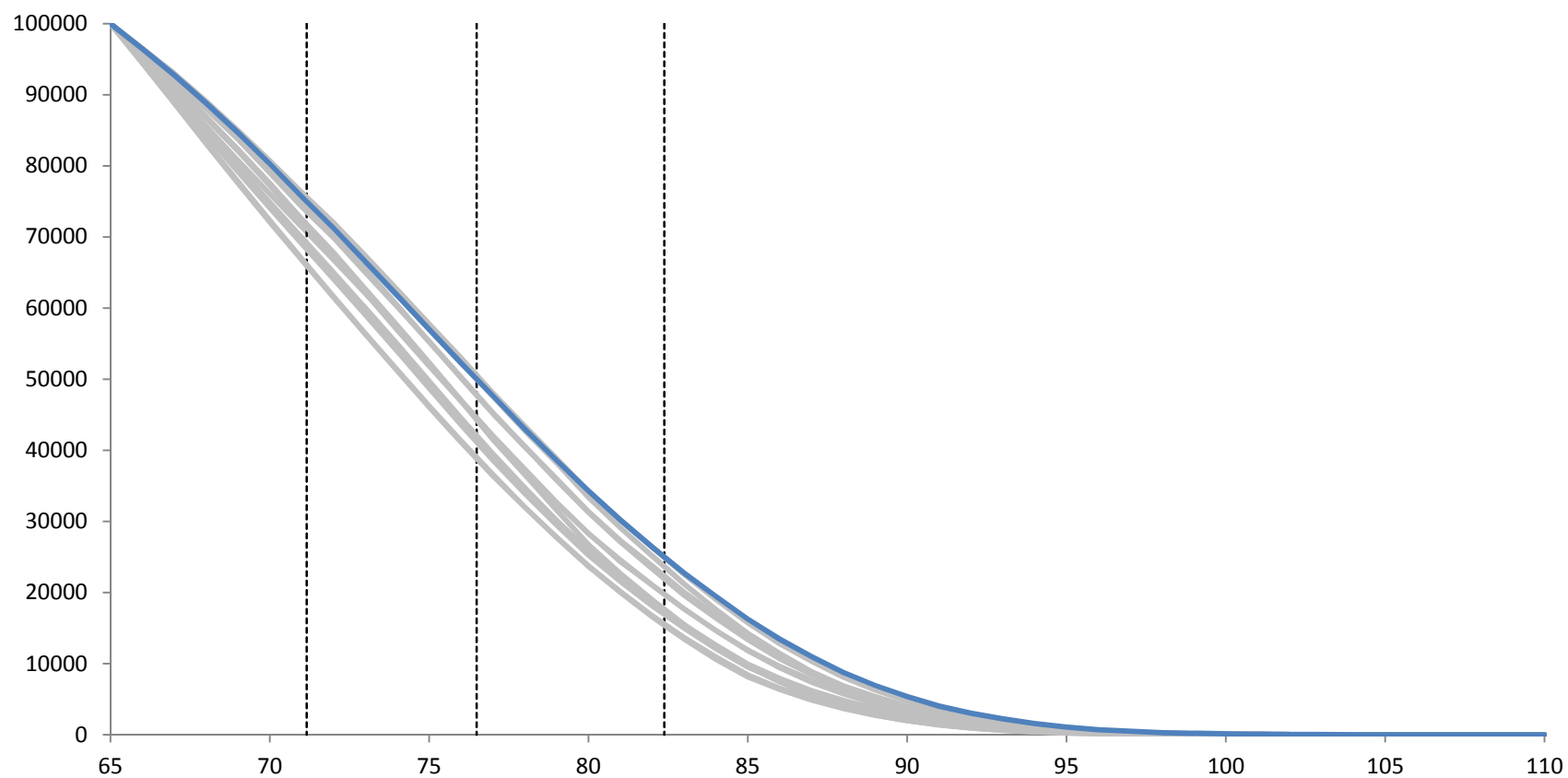
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Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

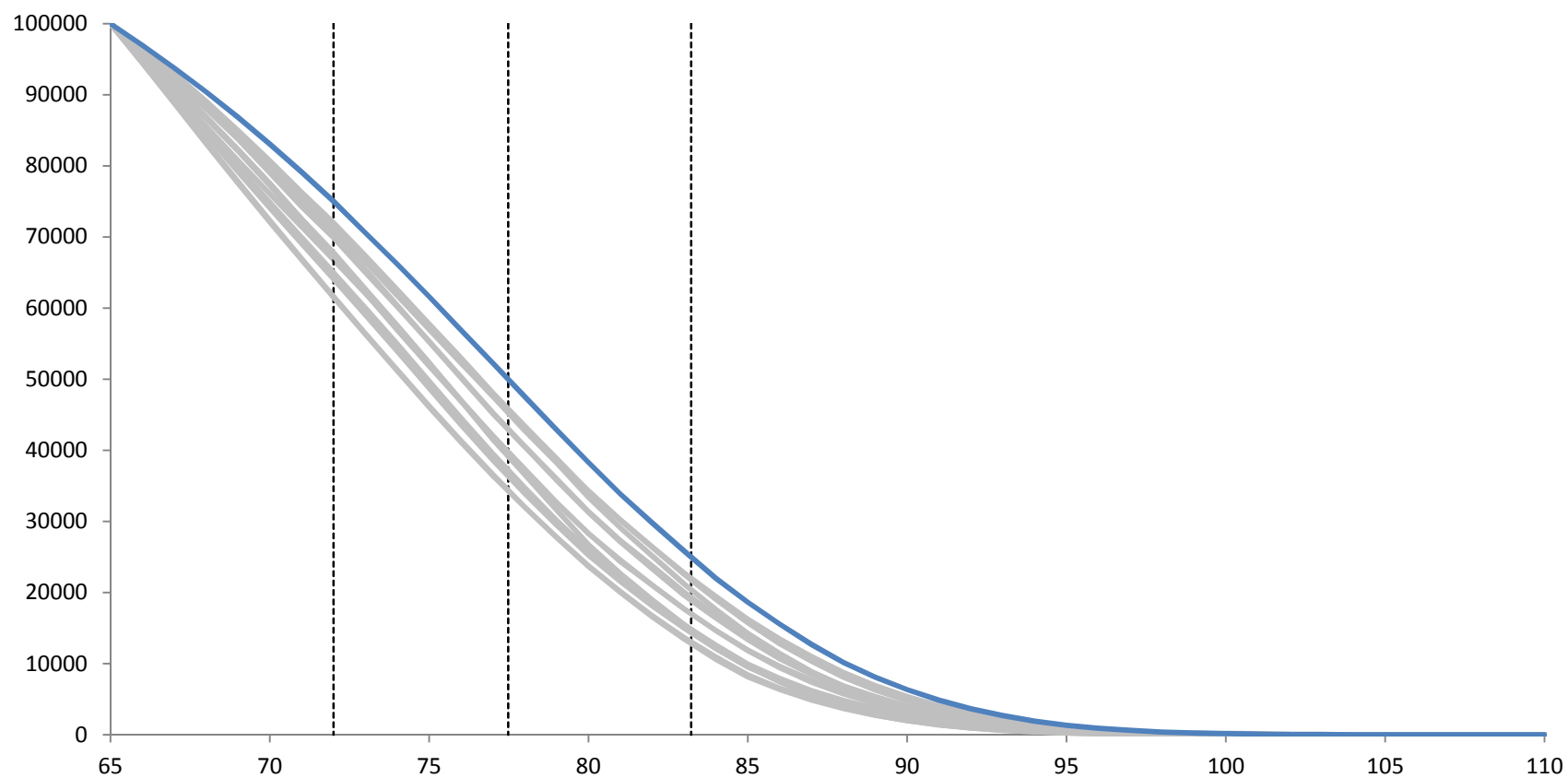
1970



Source: Human Mortality Database

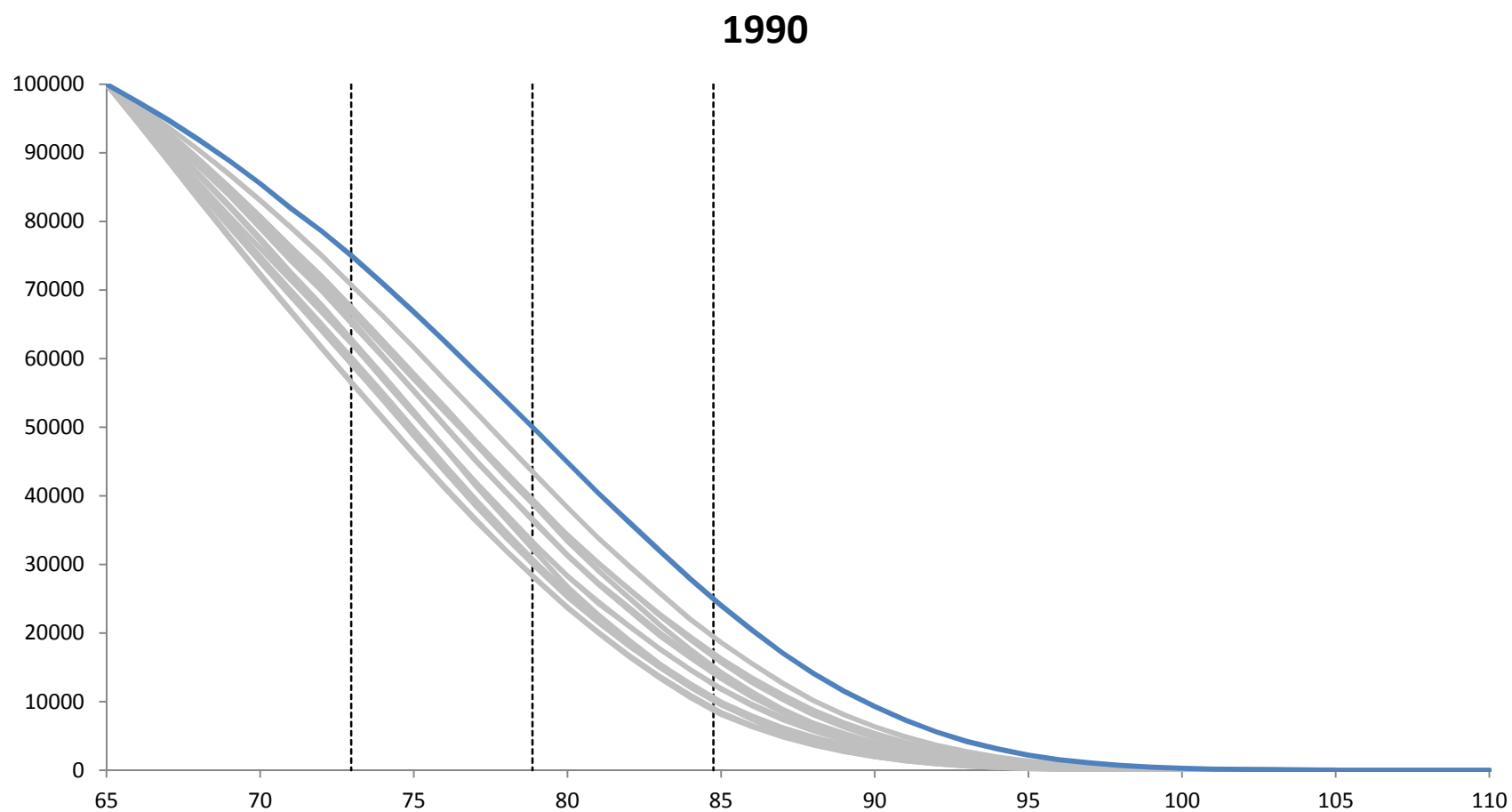
Male number of survivors conditional on reaching age 65, England and Wales 1850-2009

1980



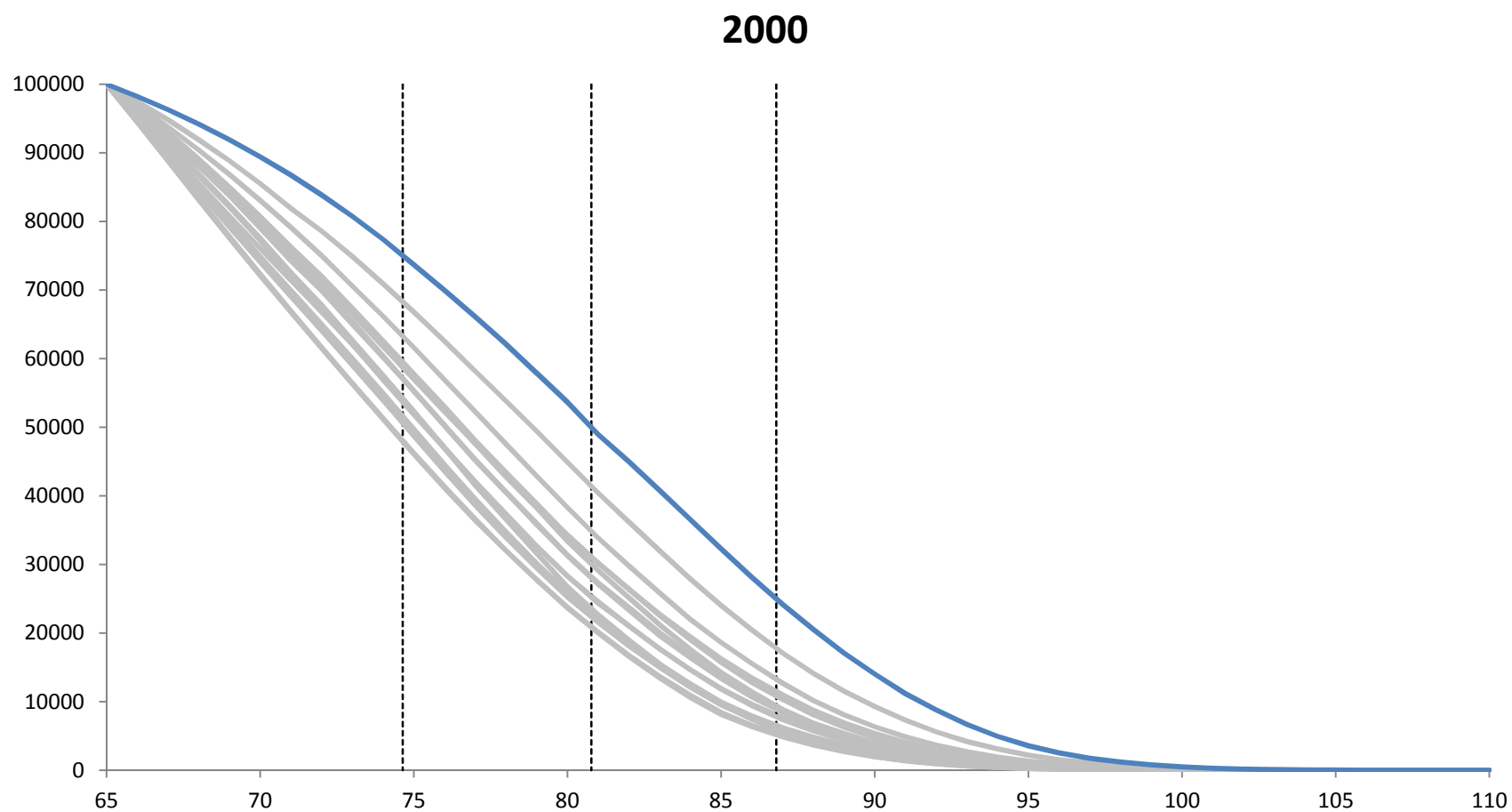
Source: Human Mortality Database

Male number of survivors conditional on reaching age 65, England and Wales 1850-2009



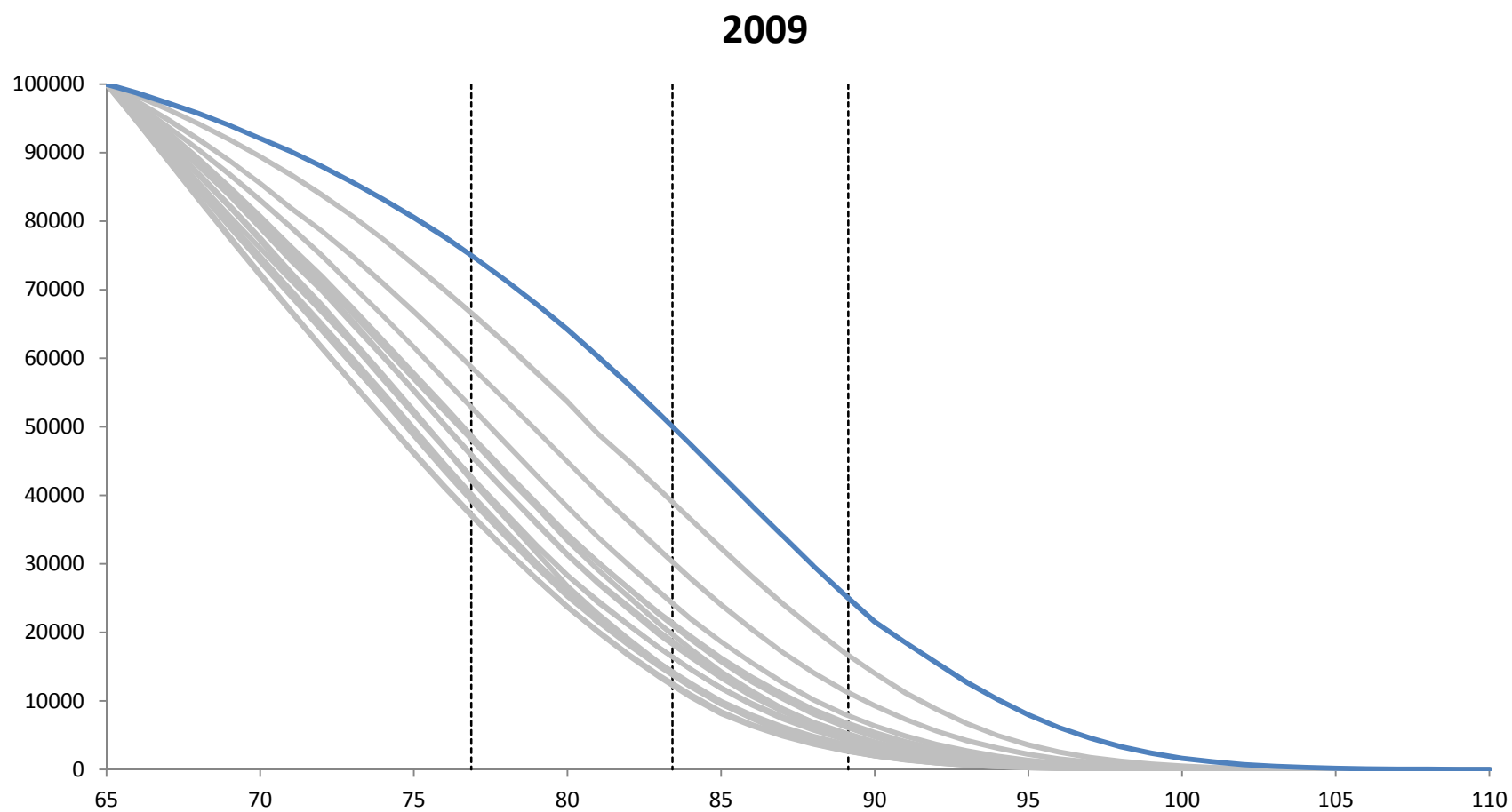
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Source: Human Mortality Database

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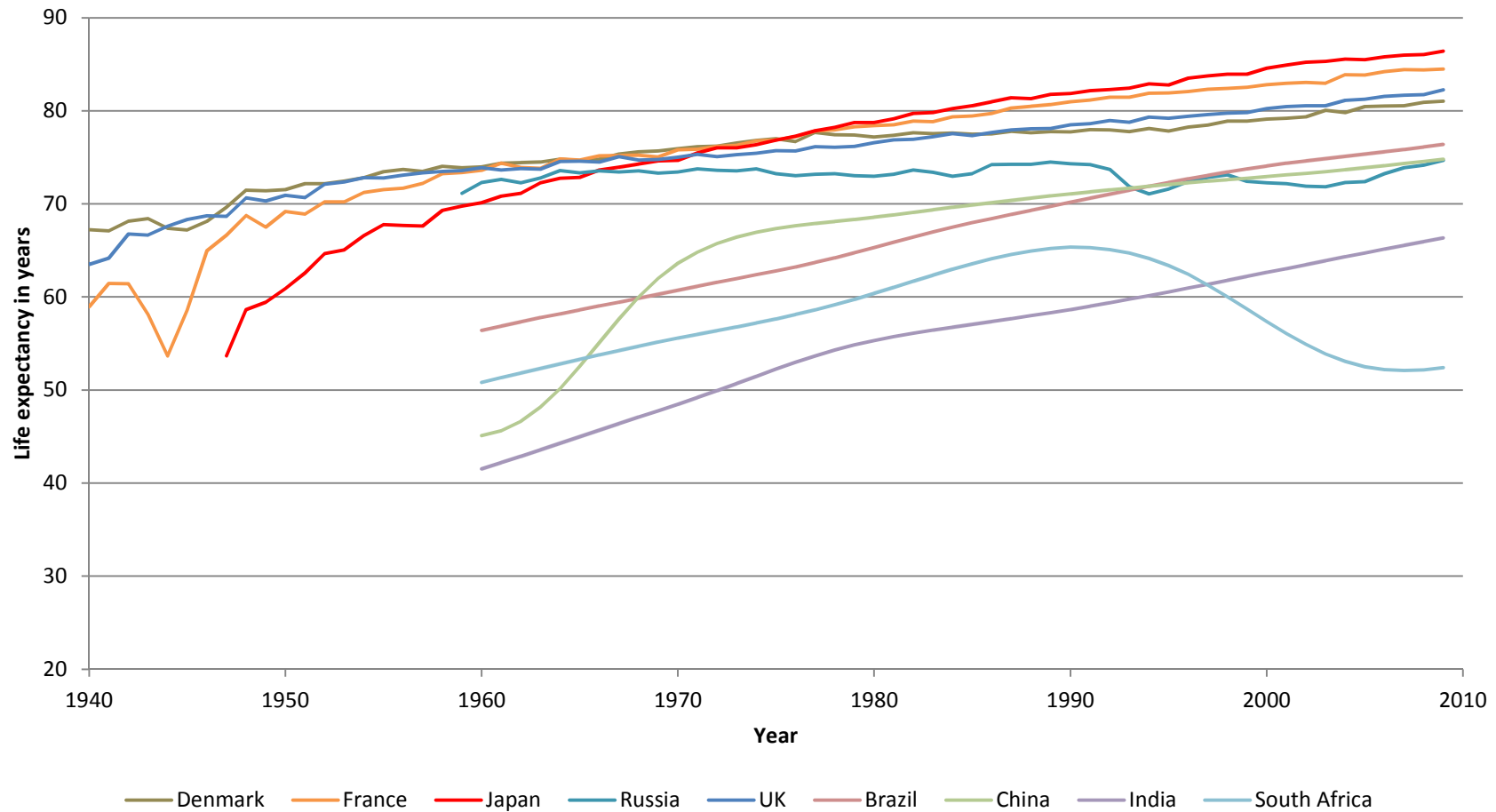
Source: Human Mortality Database

DISTRIBUTION OF AGES AT DEATH CONDITIONAL ON REACHING AGE 65 (ENGLAND AND WALES)

	Males		Females	
Year	Median	IQR	Median	IQR
1851	75.0	70.1-80.1	75.8	70.6 – 81.5
1871	74.8	70.0 – 80.3	75.7	70.6 – 81.4
1891	74.6	69.8 – 80.0	75.6	70.5 – 81.2
1911	75.3	70.4 – 80.8	76.9	71.5 – 82.4
1931	75.8	70.8 – 81.1	77.8	72.4 – 83.3
1951	76.3	71.1 – 81.7	79.3	73.7 – 84.7
1971	76.6	71.2 – 82.4	81.2	75.1 – 86.9
1991	79.0	72.0 – 85.0	83.5	76.7 – 89.5
2001	81.1	72.9 – 87.0	84.7	78.1 – 90.4

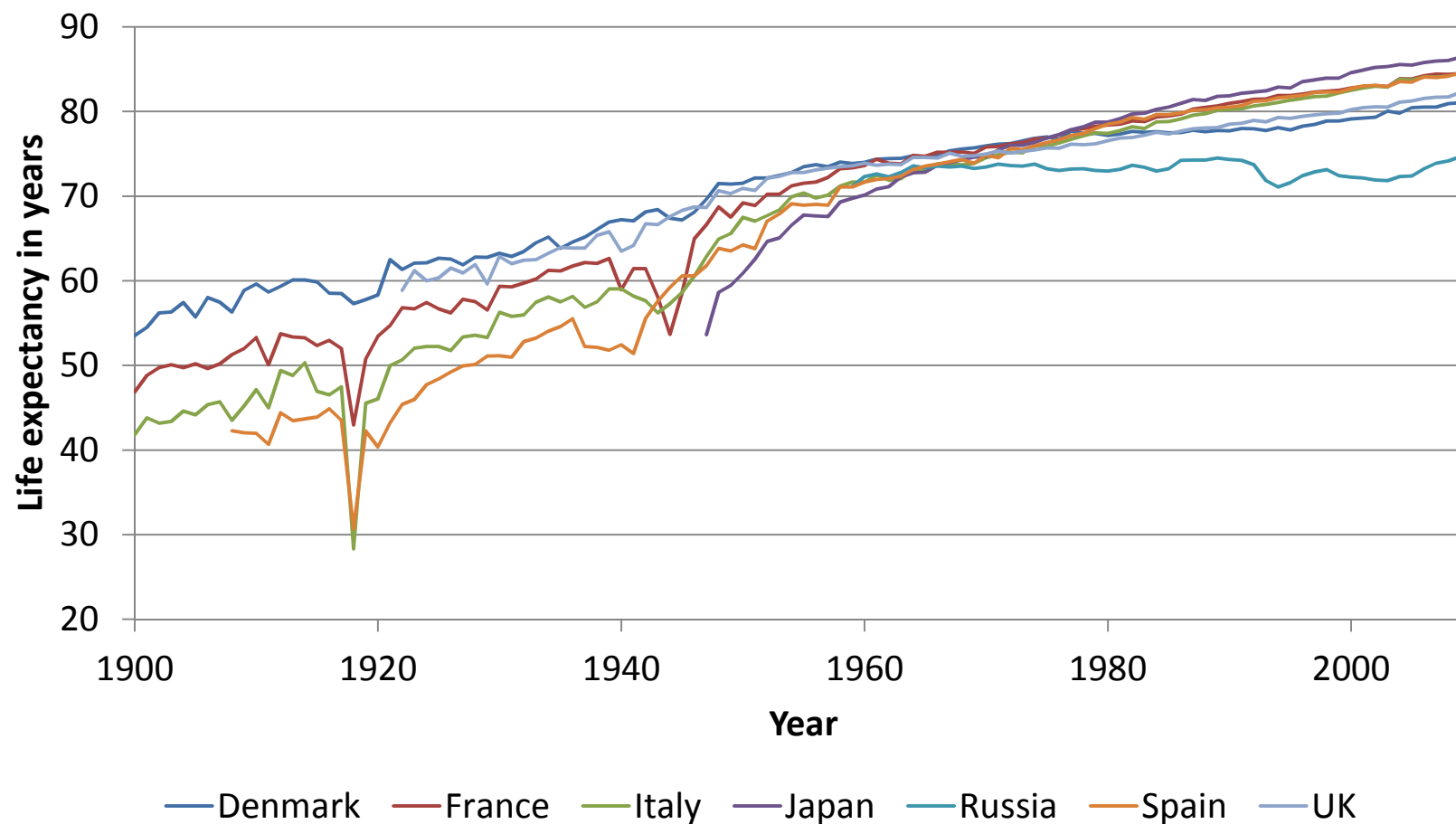
**ARE THESE TRENDS HAPPENING IN
ALL COUNTRIES?**

Female period life expectancy at birth



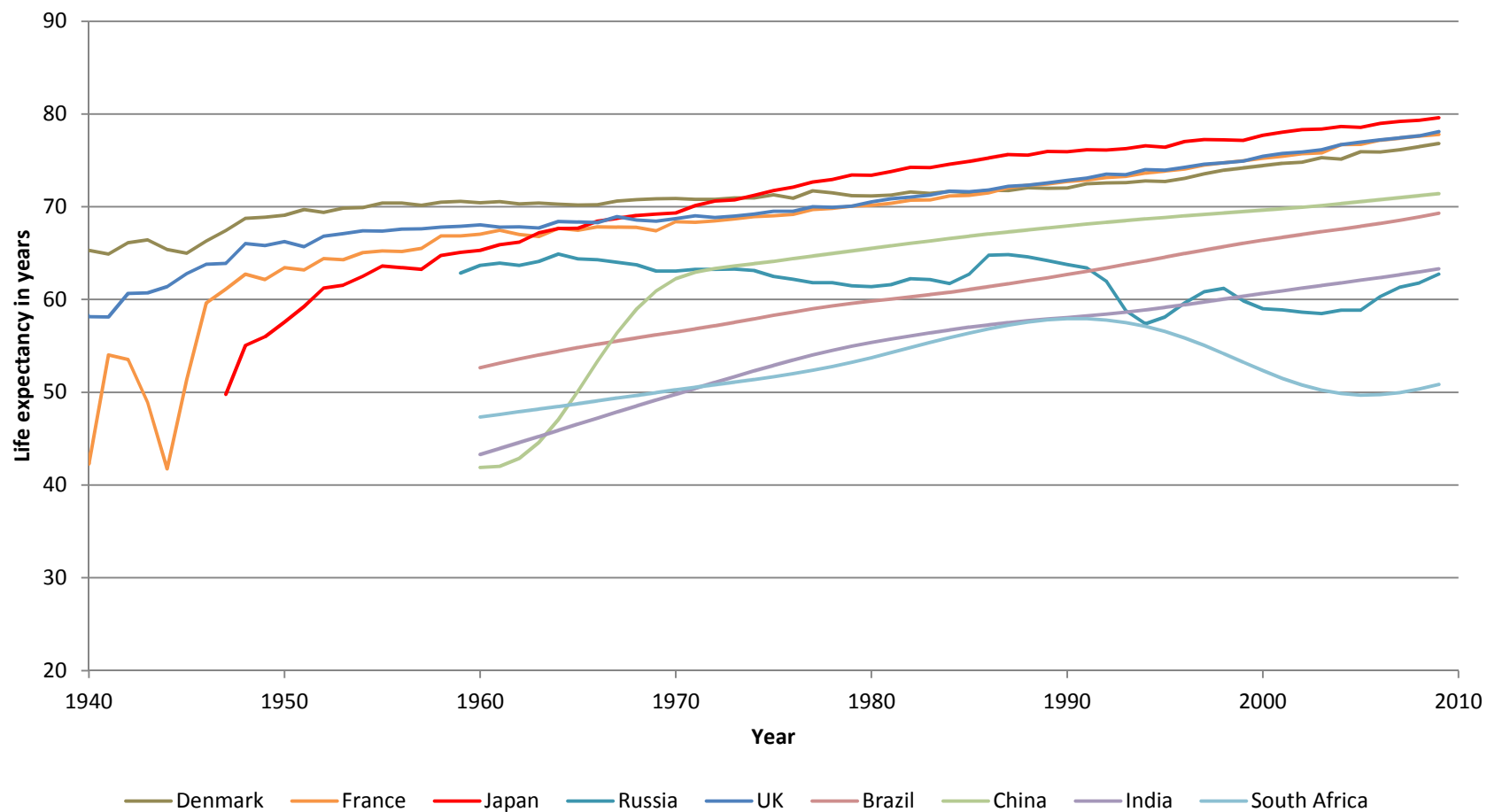
Source: Human Mortality Database and World Bank Database

Female period life expectancy at birth



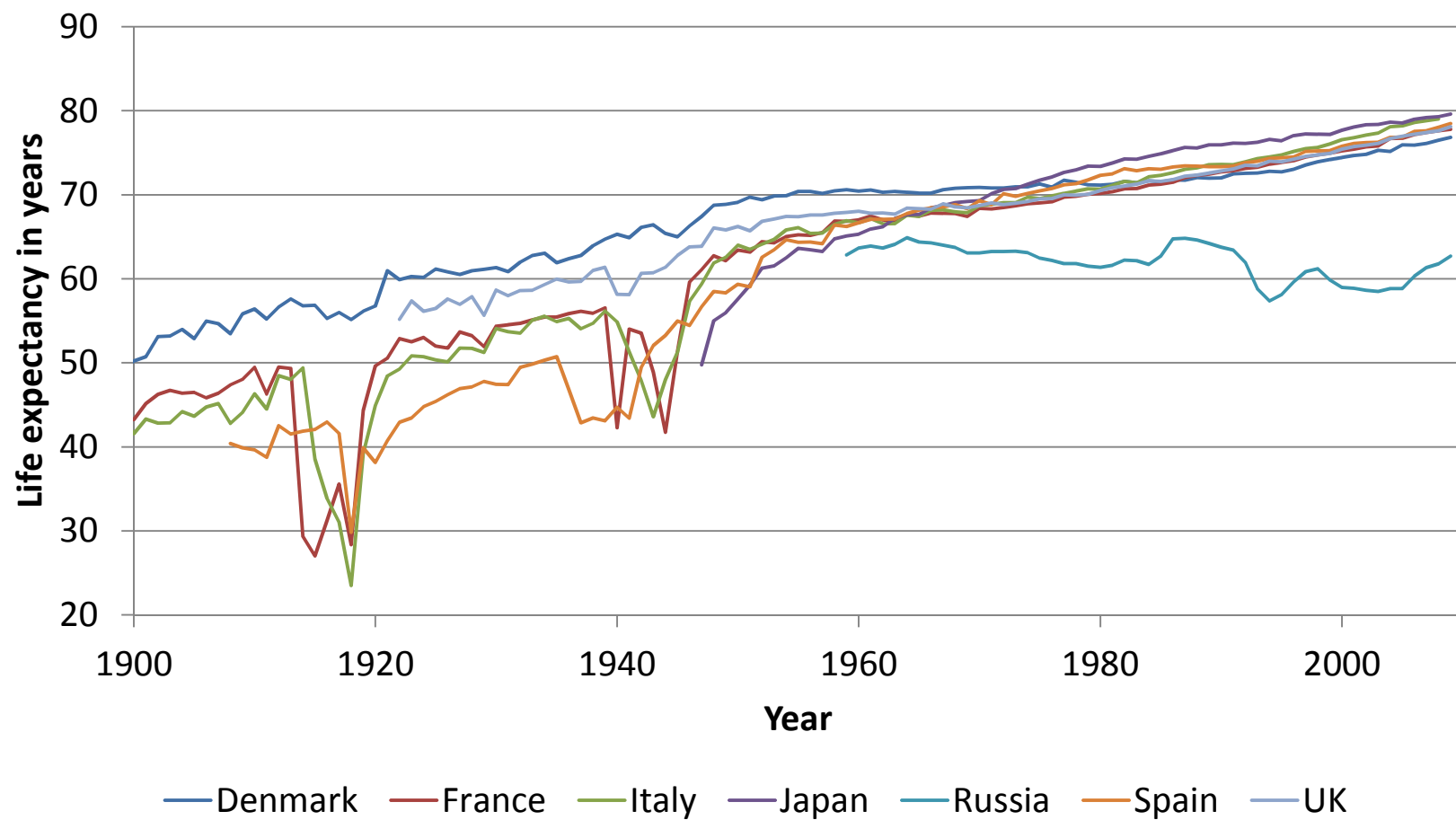
Source: Human Mortality Database

Male period life expectancy at birth



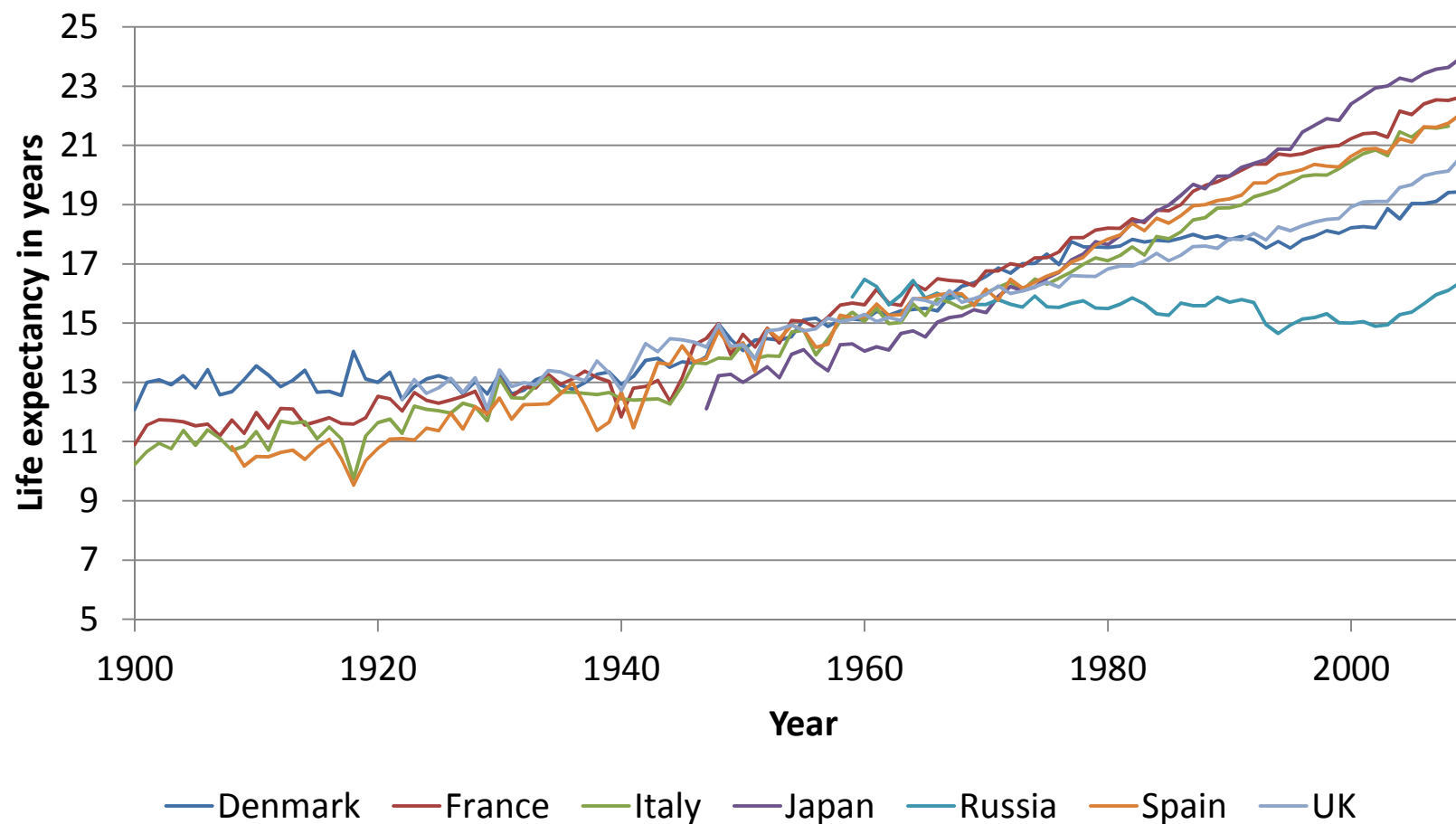
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Male period life expectancy at birth



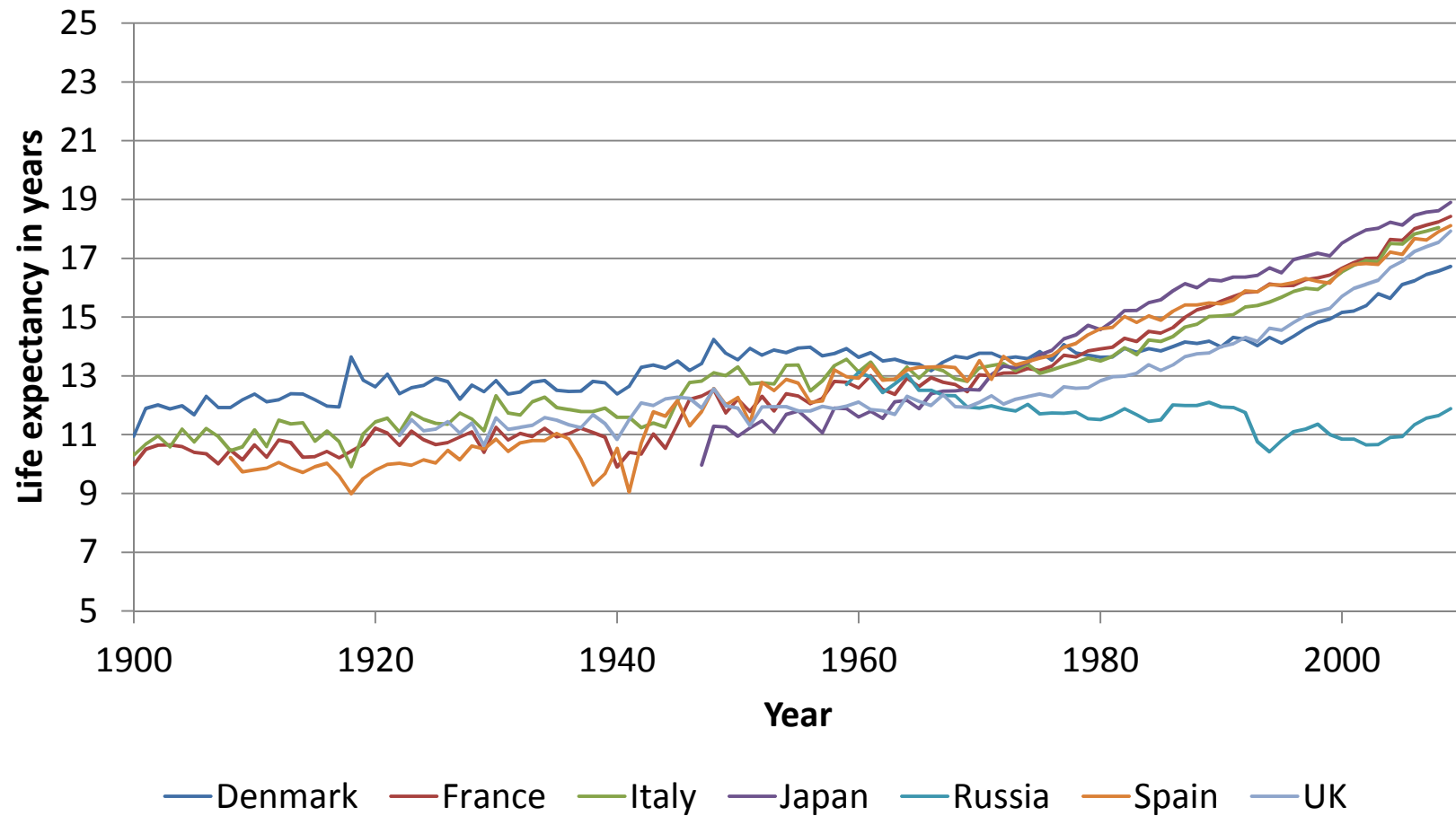
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Female period life expectancy at age 65



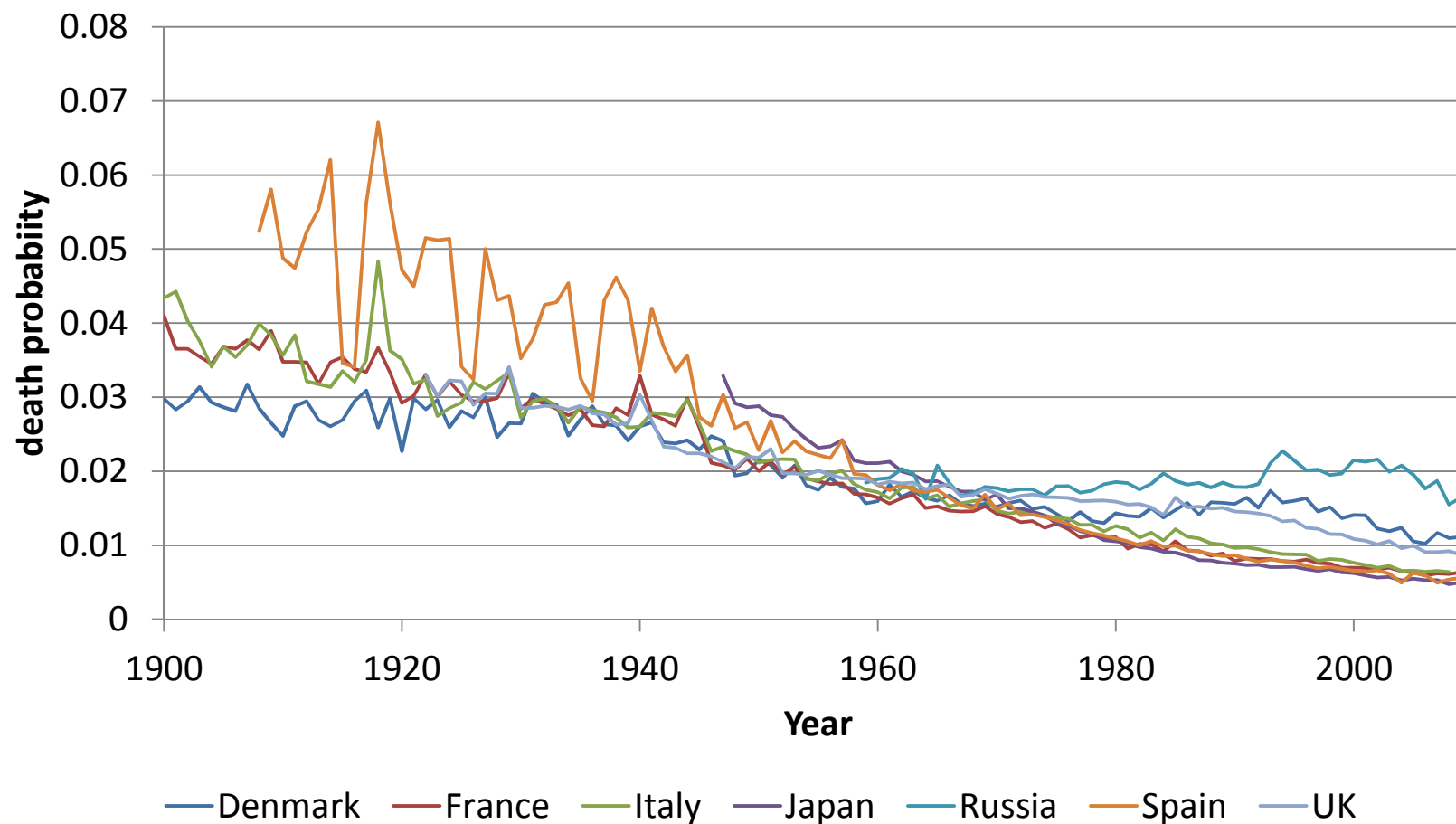
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Male period life expectancy at age 65



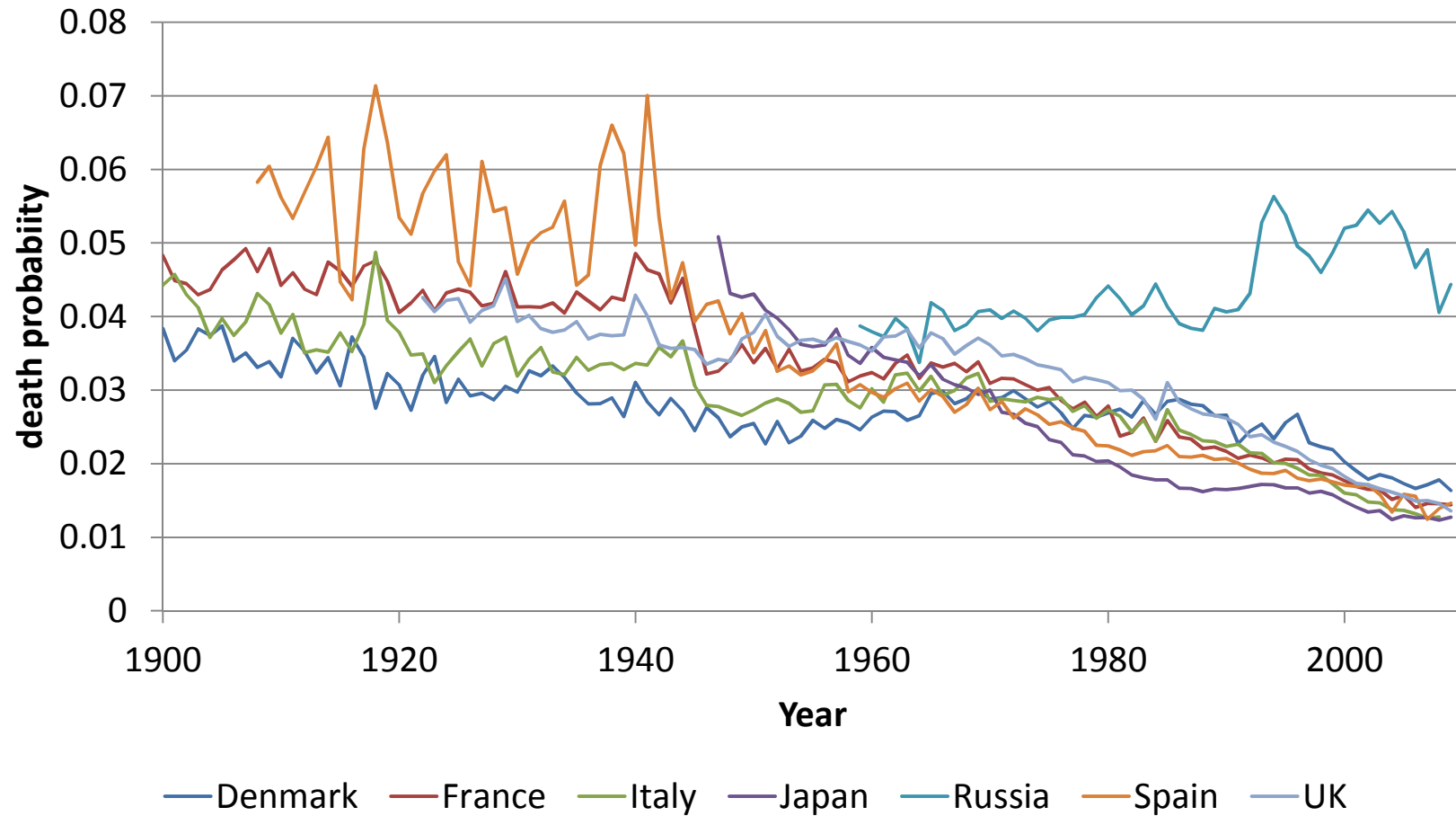
Source: Human Mortality Database

Female death probability at age 65



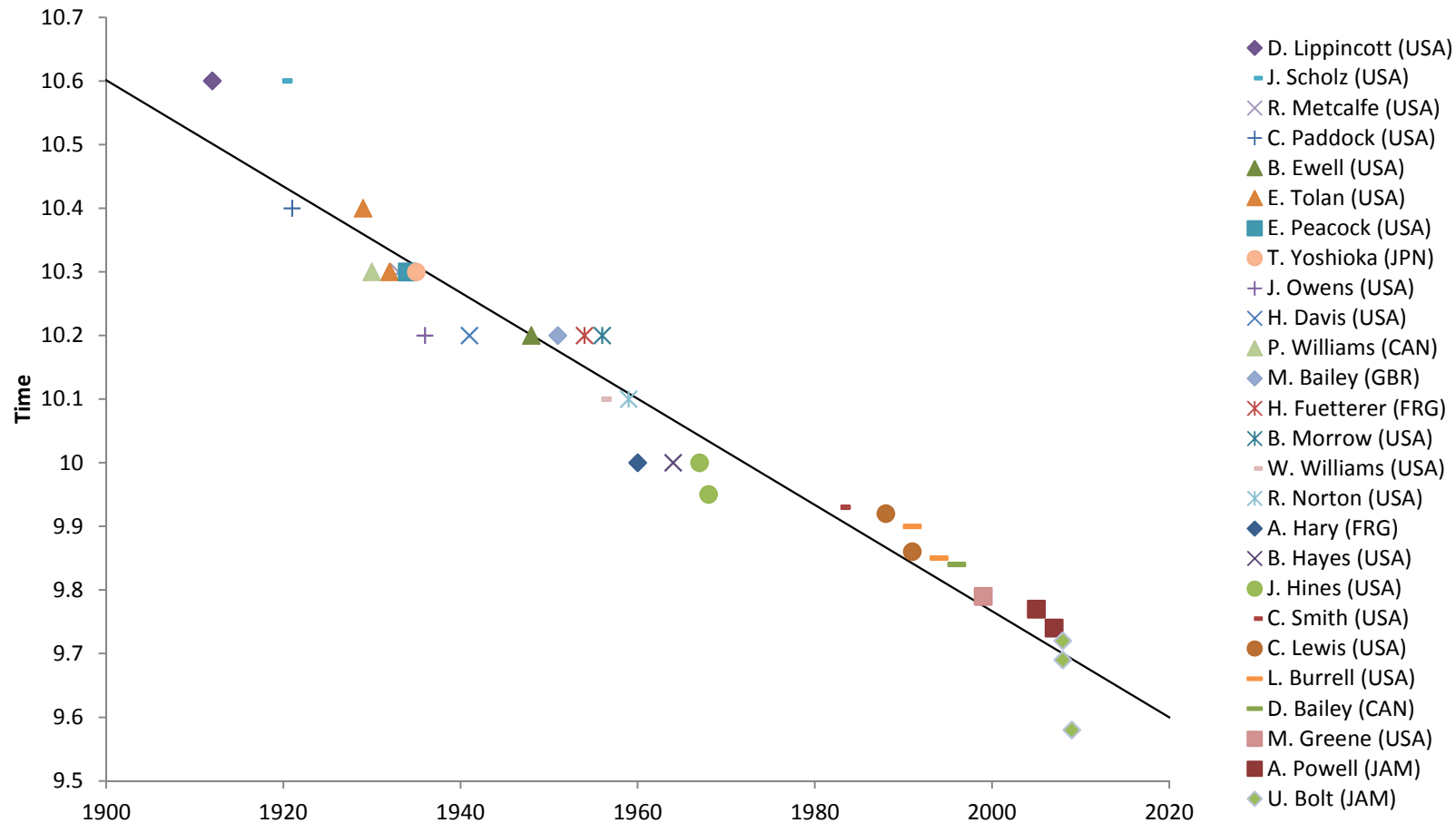
Source: Human Mortality Database

Male death probability at age 65



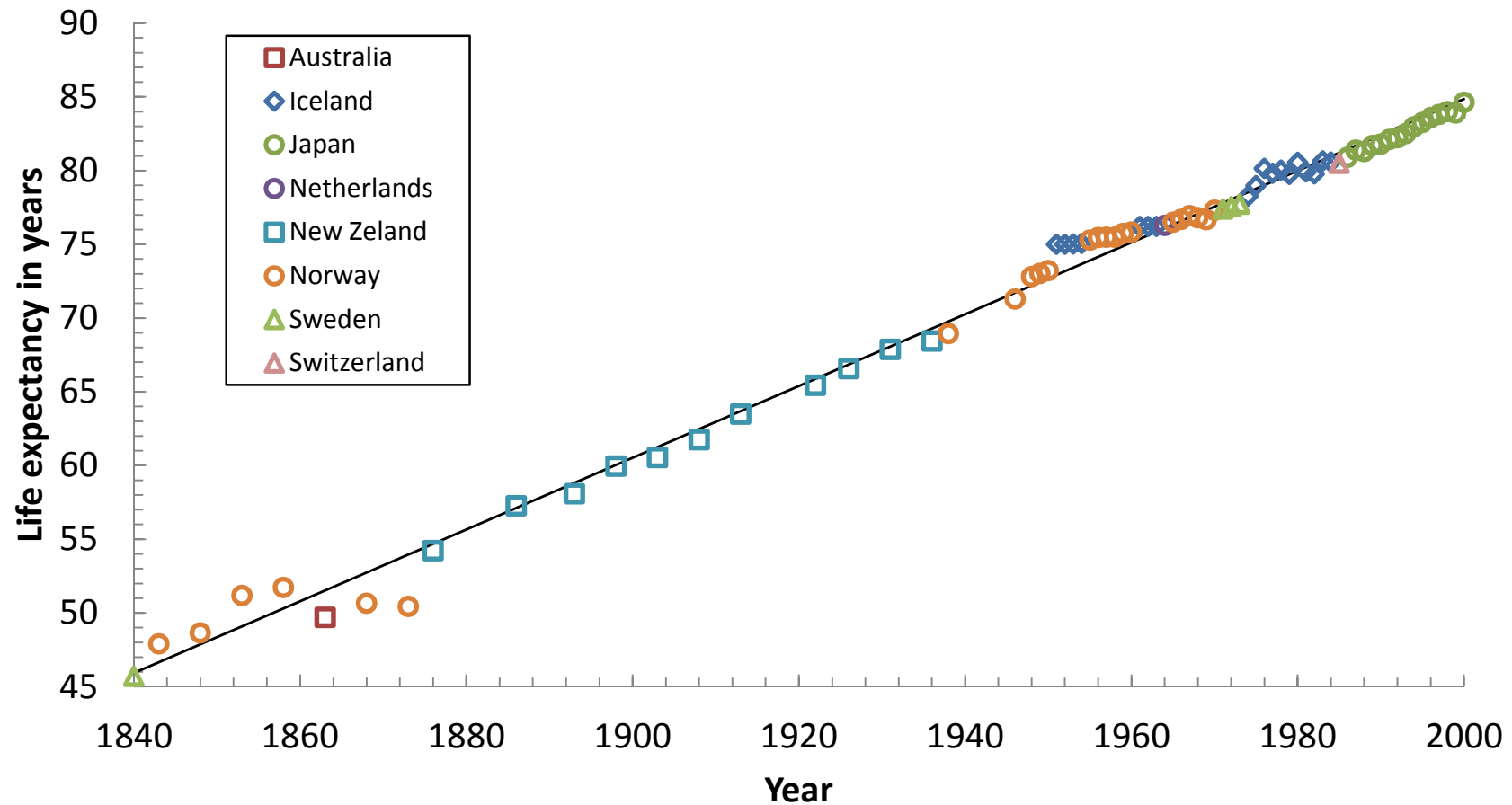
Source: Human Mortality Database

Male 100 Metres World Record



Source: IAAF

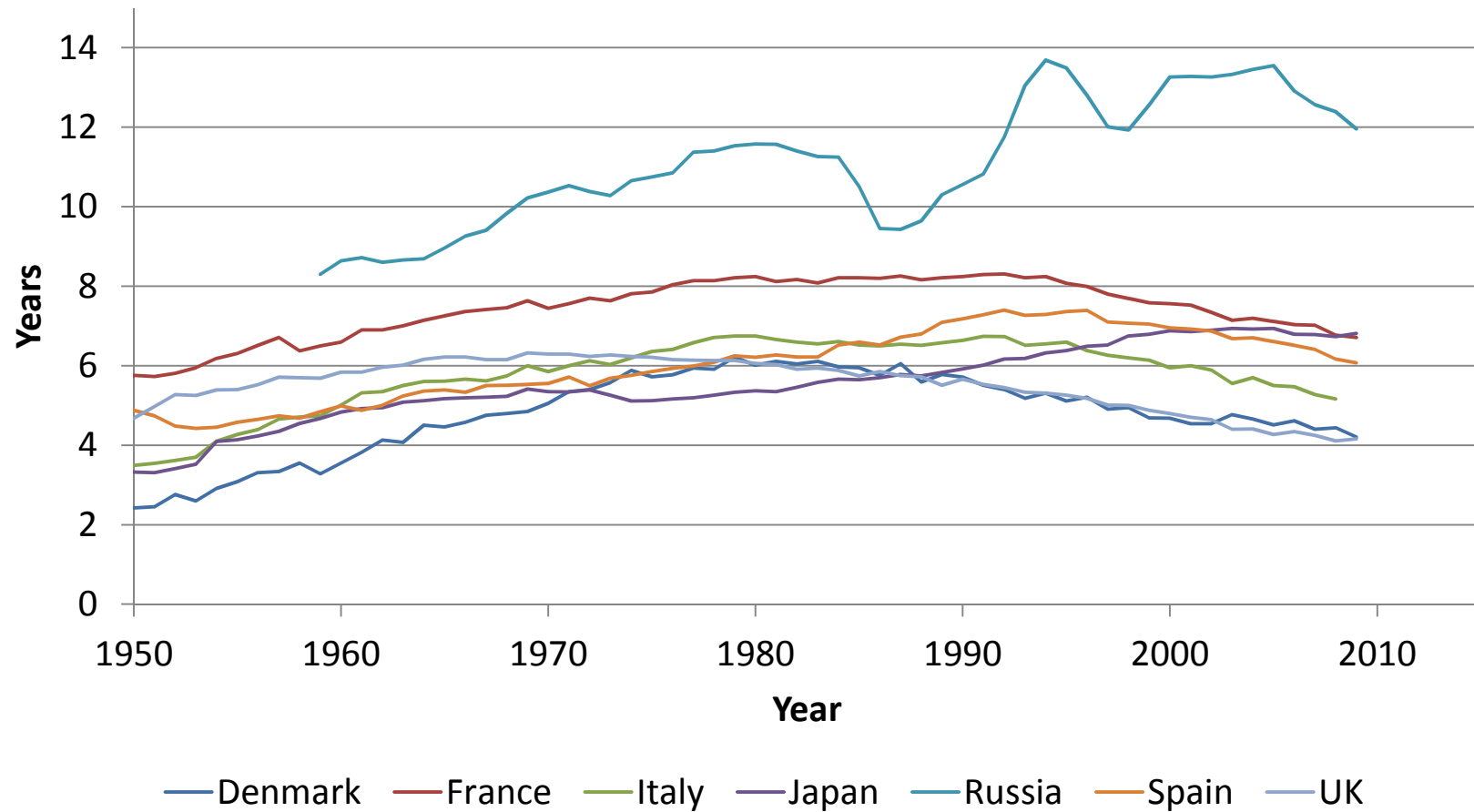
Record female life expectancy



Source: Supplementary material Oeppen and Vaupel (2002)

**ARE THEY THE SAME FOR BOTH
GENDERS?**

Gender difference in life expectancy at birth



Source: Human Mortality Database

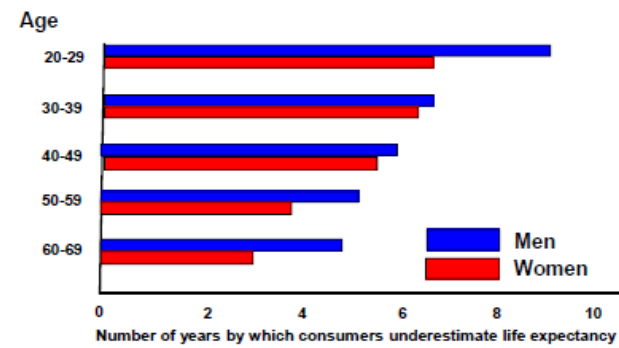
FINANCIAL BACKGROUND

- Up to 2000, high equity returns hid the problem
- Since 2000, poor equity returns and low interest rates: longevity improvements became a problem

FINANCIAL VIEW OF THE PROBLEM

- Trends in underlying mortality rates are uncertain: mixture of short term fluctuations and long term trends.
- Systematic underestimation of how long people are going to live: future trends are difficult to predict.
- Dangers
 - Individuals outlive their saving:
 - As baby boomers retire, decumulation and longevity risk become important.
 - Defined pension plans guarantee retirement income for however long people live:
 - Plan sponsors risk diverting resources away from dividend and investment programmes.
 - Annuity providers have inadequate reserves.

Individual underestimates of life expectancy by age



Sources: O'Brien, et al. (2005), self-estimated life expectancy compared with GAD forecast life expectancy

WHAT DO WE NEED FOR MANAGING LONGEVITY RISK?

- Analysis of causal factors underlying longevity.
- Analysis of ageing process.
- Quantifying longevity risk:
 - Mortality indices.
 - Stochastic mortality forecasting models – central forecasts and measures of uncertainty.

NEED FOR STOCHASTIC MORTALITY MODELS

- Risk management.
- Setting reserves.
- Solvency II capital requirements
- Contracts with embedded options e.g. variable annuities
- Pricing and hedging mortality-linked securities e.g. longevity swaps

MEASURING AND QUANTIFYING LONGEVITY RISK

ALTERNATIVE EXPERT VIEWS: THE EXTREMES

- ‘Pessimists’ suggest that life expectancy might level off or decline (Olshansky)
 - Impact of obesity, poor diet, global warming etc.
- ‘Optimists’ suggest no natural limit to human life (Vaupel).
 - Supported by extrapolative methods.
 - Future scientific advances?

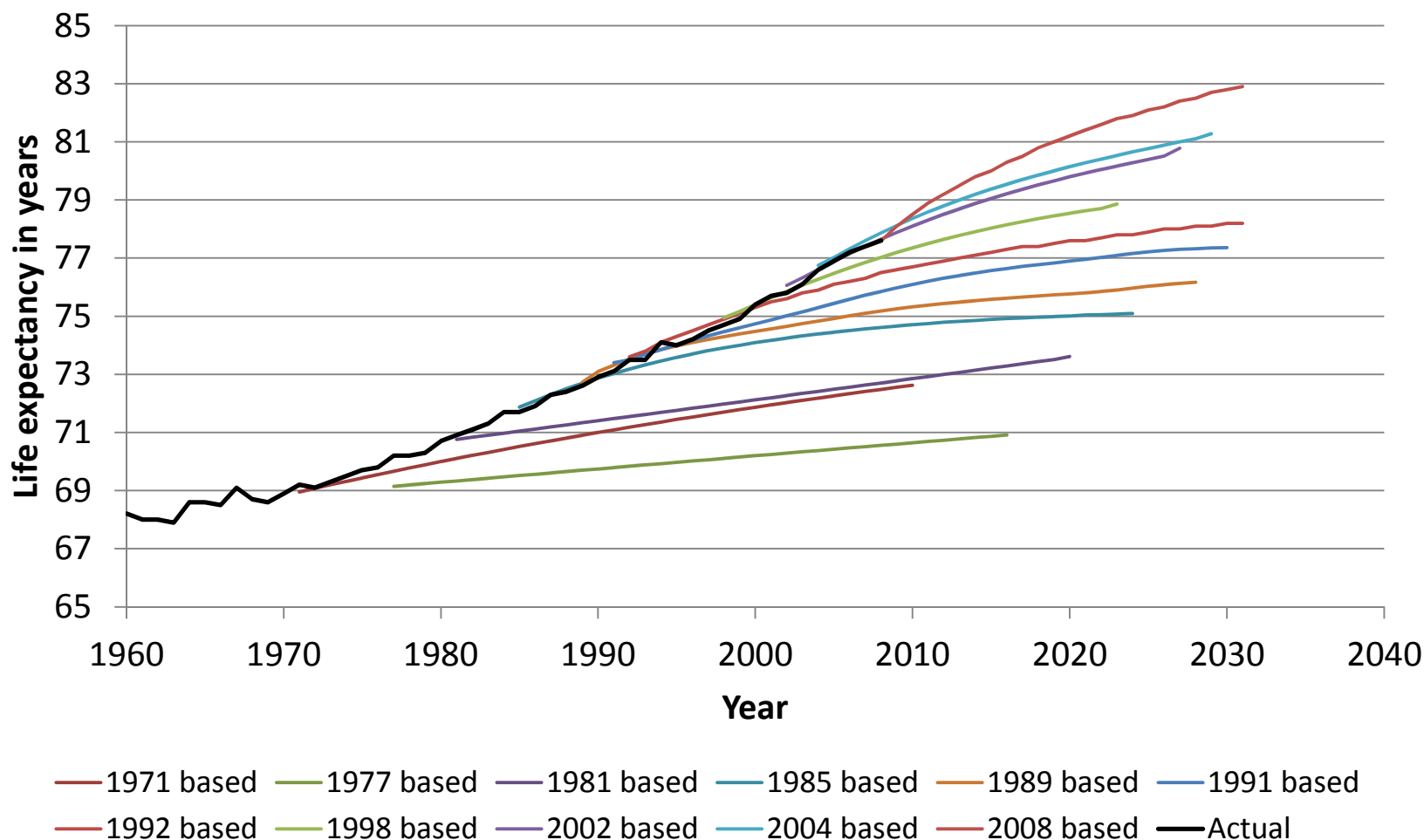
MORTALITY FORECASTING METHODOLOGIES

- Expert based.
- Structural Modelling (Explanatory or Econometric).
- Decomposition.
- Trend Modelling (Extrapolation).

REFLECTIONS

- Extrapolation methods fail to account for future structural change.
- Expert opinion has been conservative e.g. choice of target, target date, interpolation path.
- Theoretical advantages of structural models not matched by forecasting performance.
- Decomposition by cause of death has led to conservative forecasts.

Accuracy of Office of National Statistics Mortality assumption (Actual and projected UK male period life expectancy at birth)



Source: Government Actuary's Department

JUSTIFICATION FOR EXTRAPOLATION METHODS

- Complexity and stability of historical trends.
- Extrapolation may be the most reliable approach in terms of forecast accuracy.
- “...we cannot afford to be ashamed of extrapolating the observed regularities of the past” (Keyfitz, 1982).

EXTRAPOLATION METHODS: SIGNIFICANT DEVELOPMENTS

- Lee-Carter model 1992
- Age-period-cohort version of LC model 2006 (Renshaw & Haberman)
- Blake-Cairns-Dowd models 2008
- Extensions based on Plat 2009

LEE-CARTER MODEL (Lee and Carter 1992)

$$\mu_x(t) = \exp(\alpha_x + \beta_x \kappa_t + \varepsilon_{xt})$$

- Parameter constraints: $\sum_x \beta_x = 1$ and $\kappa_{t_1} = 0$
- Interpretation of parameters
- Methods of fitting: SVD, WLS, GLM formulation
- Diagnostics
- Smoothing of β_x
- Forecasting: based on time series models of κ_t

Also fitted to $\log \left[\frac{q_x(t)}{1 - q_x(t)} \right]$

EXTENSIONS TO IMPROVE FIT

- Optimize choice of fitting period
- Add extra time factors (Renshaw and Haberman 2003)

$$\exp(\alpha_x + \beta_x^{(1)} \kappa_t^{(1)} + \beta_x^{(2)} \kappa_t^{(2)})$$

- Allow for cohort effect (Renshaw and Haberman 2006)

APC VERSION OF LEE-CARTER

$$\mu_x(t) = \exp(\alpha_x + \beta_x^{(1)}\kappa_t + \beta_x^{(0)}i_z + \varepsilon_{xtz}) \text{ where } z = t - x$$

Note strong observed cohort effect for particular cohorts in UK (1925-45), US, France, Germany, Japan, Sweden.

Parameter constraints: $\sum_x \beta_x^{(1)} = 1, \sum_x \beta_x^{(0)} = 1, \kappa_{t_1} = 0, \beta_x^{(1)} > 0.$

Also fitted to $\log \left[\frac{q_x(t)}{1 - q_x(t)} \right]$

LEE-CARTER MODEL STRUCTURES INVESTIGATED

$$\text{LC} \quad \alpha_x + \beta_x^{(1)} \kappa_t$$

$$\text{H1} \quad \alpha_x + \beta_x^{(1)} \kappa_t + i_z$$

$$\text{M} \quad \alpha_x + \beta_x^{(1)} \kappa_t + \beta_x^{(0)} i_z$$

$$\text{LC2} \quad \alpha_x + \beta_x^{(1)} \kappa_t^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$$

BLAKE-CAIRNS-DOWD MODELS

(Cairns et al 2008, 2009)

4 main models proposed for $\log \left[\frac{q_x(t)}{1-q_x(t)} \right]$ and “older” age
Range:

M5 $\kappa_t^{(1)} + \beta_x^{(1)} \kappa_t^{(2)}$

M6 $\kappa_t^{(1)} + \beta_x^{(1)} \kappa_t^{(2)} + i_z$

M7 $\kappa_t^{(1)} + \beta_x^{(1)} \kappa_t^{(2)} + \beta_x^{(2)} \kappa_t^{(3)} + i_z$

M8 $\kappa_t^{(1)} + \beta_x^{(1)} \kappa_t^{(2)} + \beta_x^{(3)} i_z$

MODEL DIAGNOSTICS AND DYNAMICS

- Diagnostics: residual plots based on the scaled deviance residuals
- Dynamics: based on a univariate or multivariate random walk with drift.

PREDICTION INTERVALS

Simulations approach, making full allowance for forecast error generated by multivariate random walk:

ALGORITHM: For simulation $m \left(m=1,2,\dots,N \right)$

- Sample randomly z_m^* from $mulN(O,I)$
- For $j=1,2,\dots,J$ (length of forecast)
- Compute $\kappa_{t_n+j}^* = \kappa_{t_n} + j\hat{\theta} + \sqrt{j}\hat{C} z_m^*$
- Compute $q_{x+j, t_n+j, m}^*$
- Compute key indices of interest

BASIS FOR COMPARISON OF MODELS

- Model fitting – optimize binomial deviance (likelihood).
- Map predictor structure η_{xt} to q_{xt} using the same link function.
- Model period factors, κ_t , as a multivariate random walk with drift.
- Consistent method of extrapolating to highest ages within each calendar period.
- Common approach to simulation of prediction intervals.

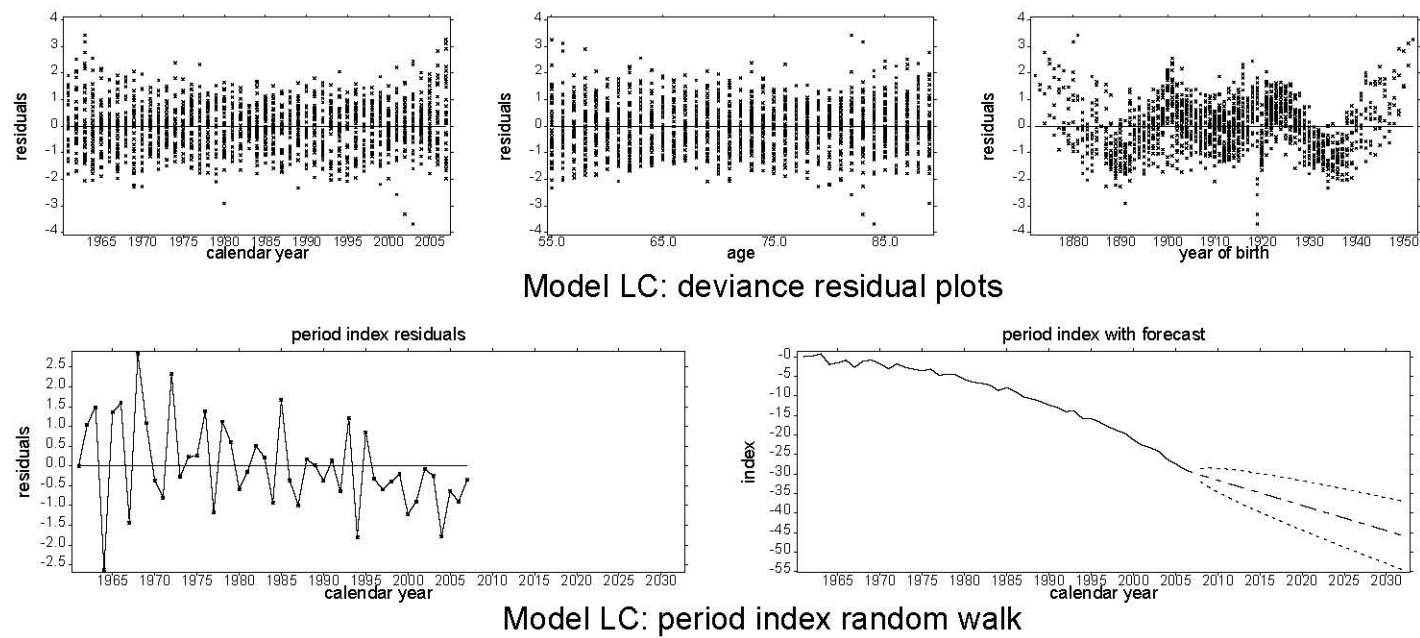
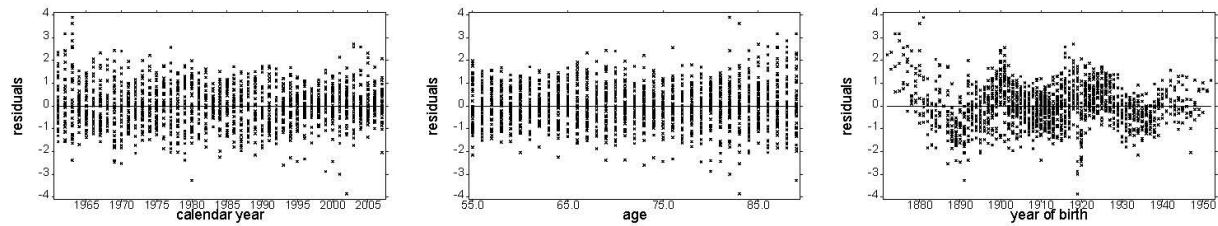
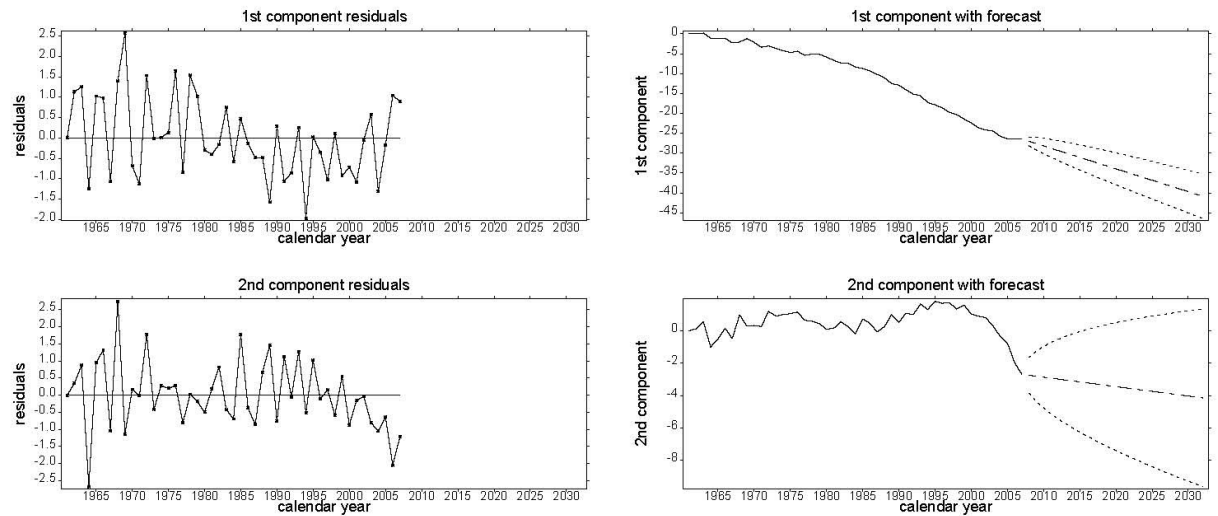


Fig 1. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor LC.



Model LC2: deviance residual plots



Model LC2: period indices bi-variate random walk

Fig 2. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor LC2.

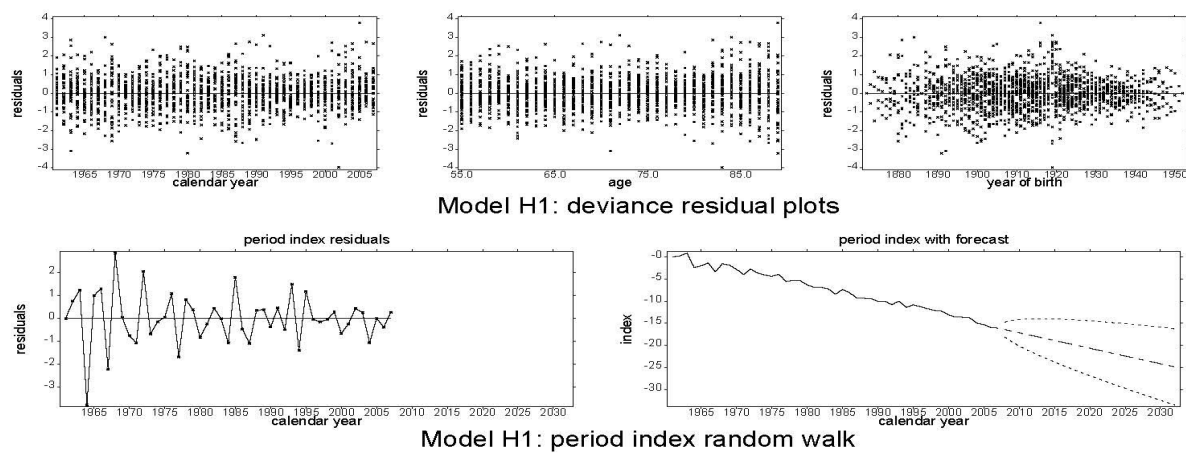


Fig 3. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor H1.

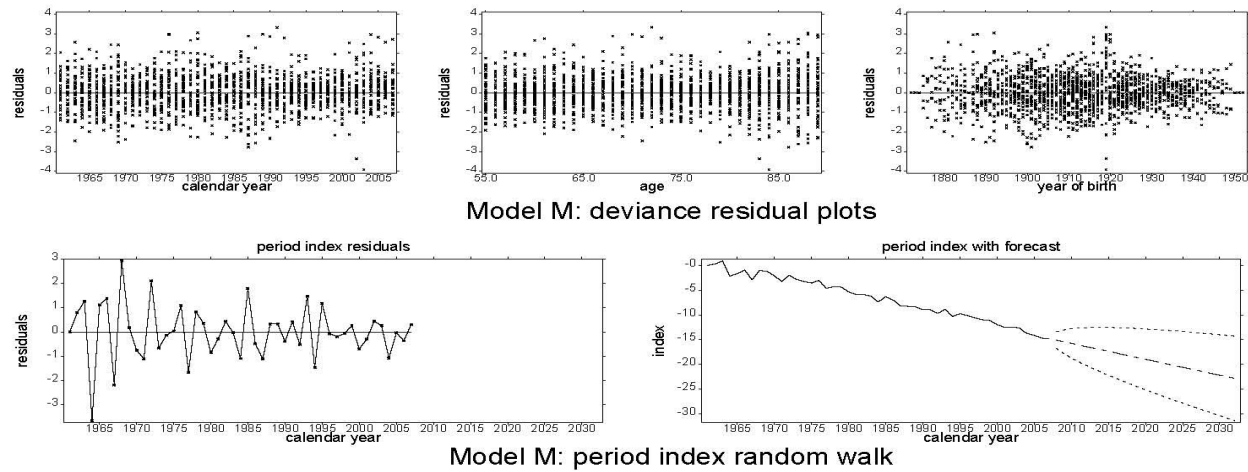
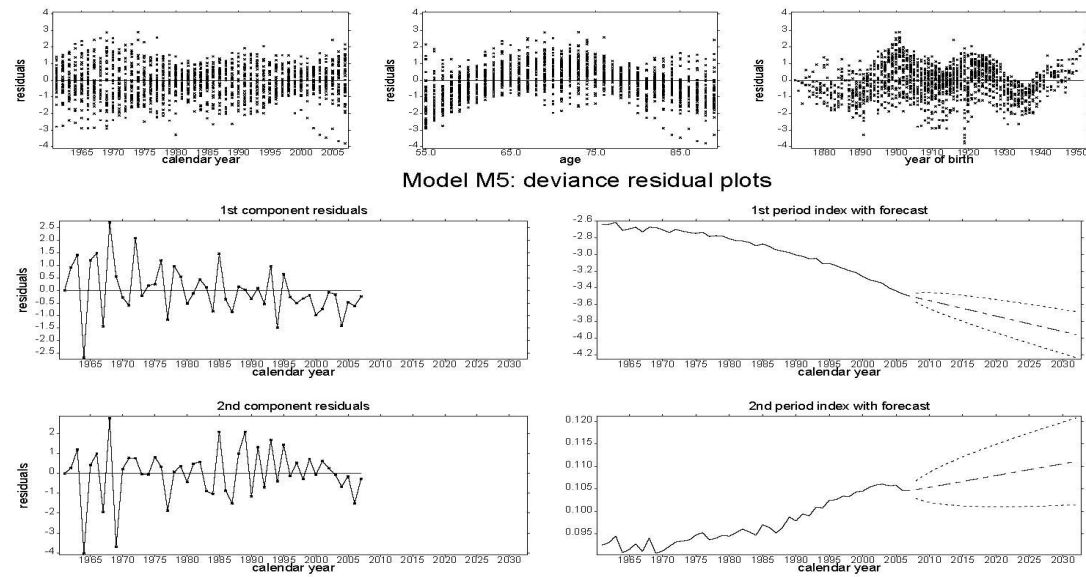
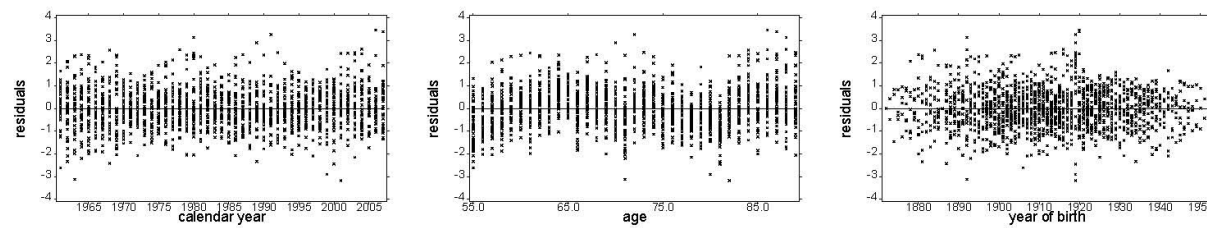


Fig 4. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor M.

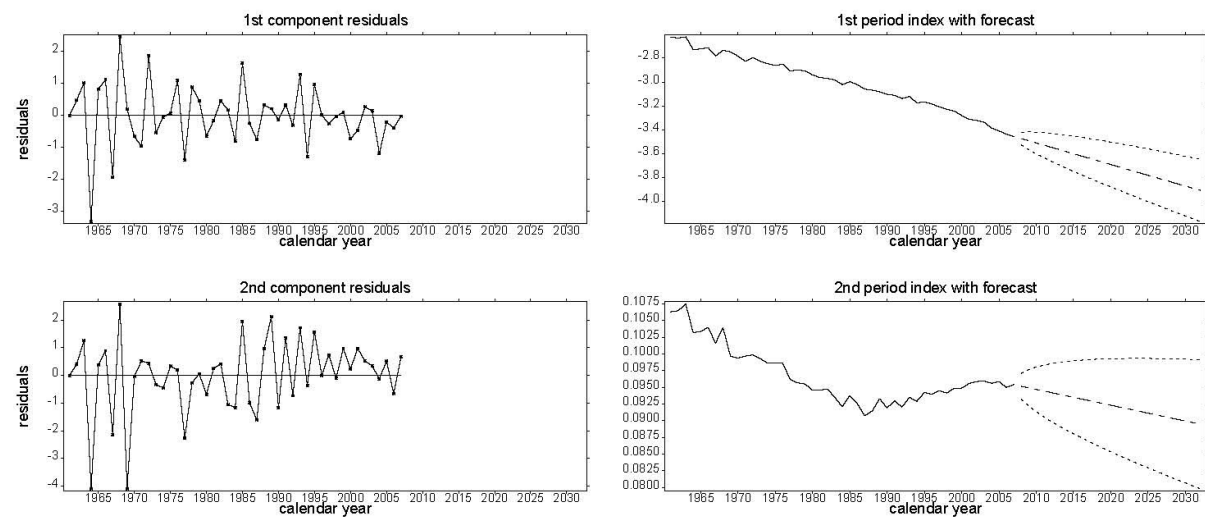


Model M5: period indices bi-variate random walk

Fig 5. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor M5.

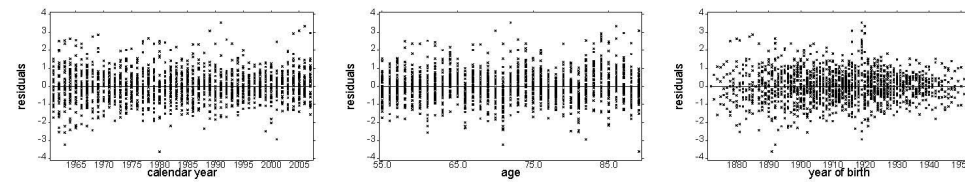


Model M6: deviance residual plots

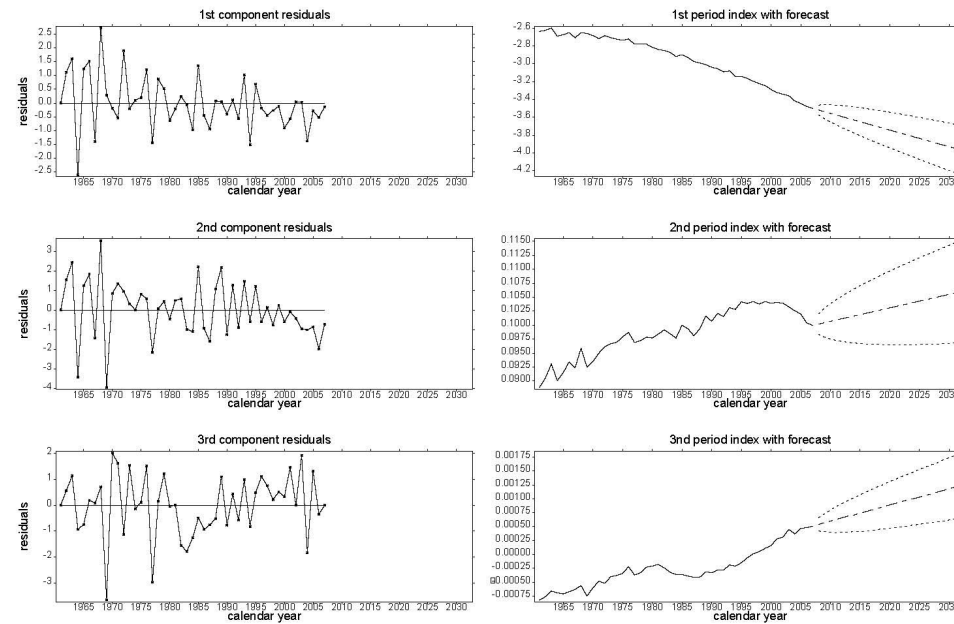


Model M6: period indices bi-variate random walk

Fig 6. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor M6.

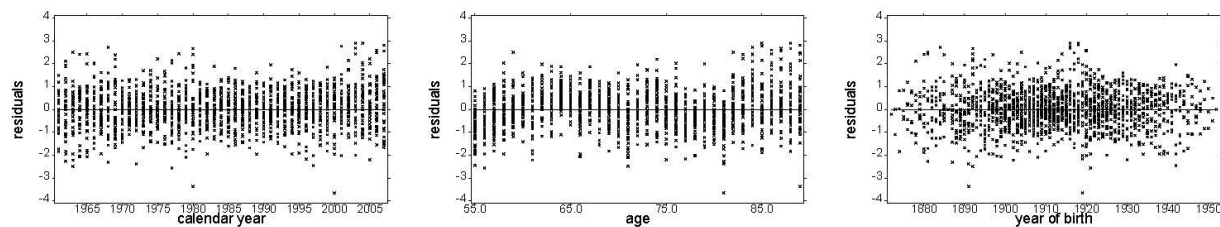


Model M7: deviance residual plots

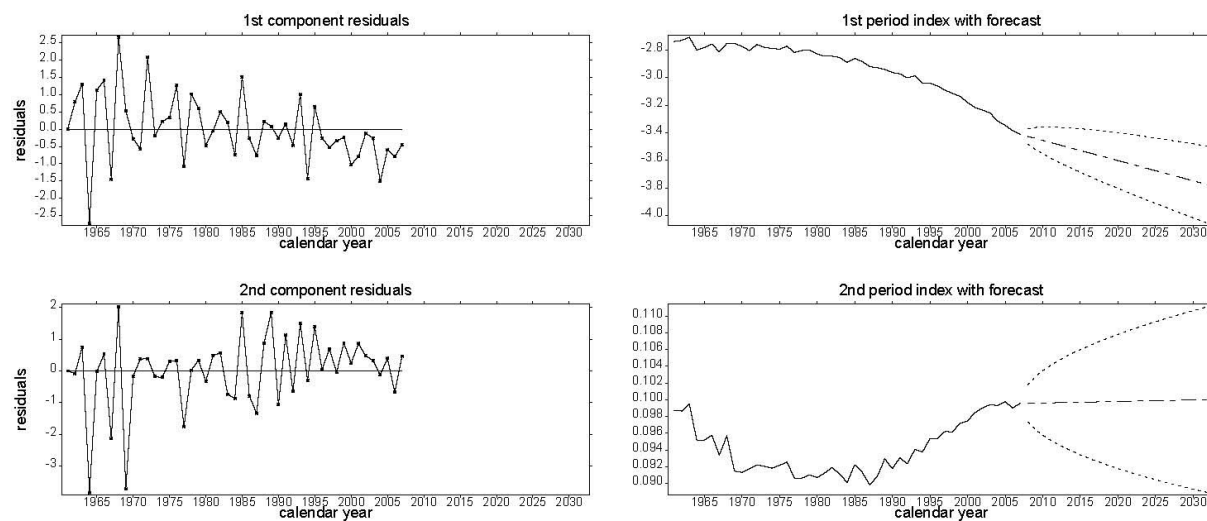


Model M7: period indices tri-variate random walk

Fig 7. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor M7.



Model M8: deviance residual plots



Model M8: period indices bi-variate random walk

Fig 8. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictor M8.

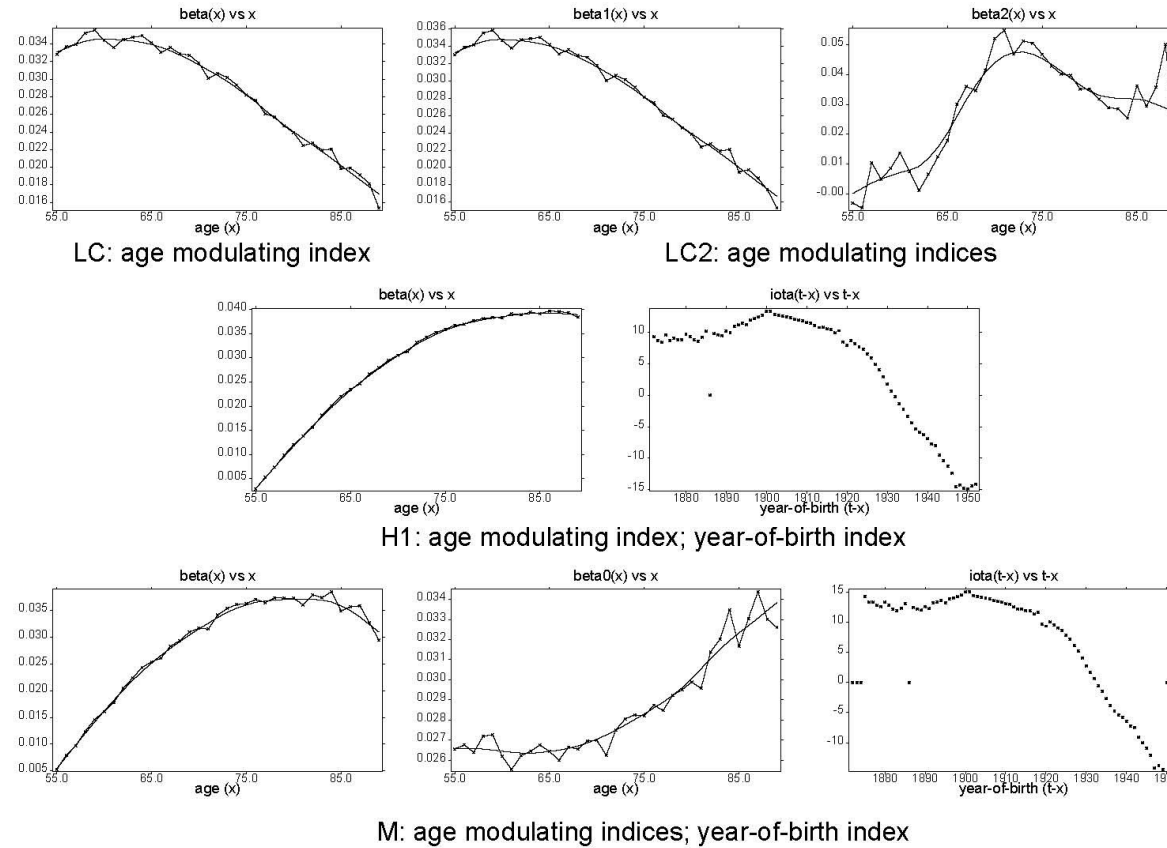
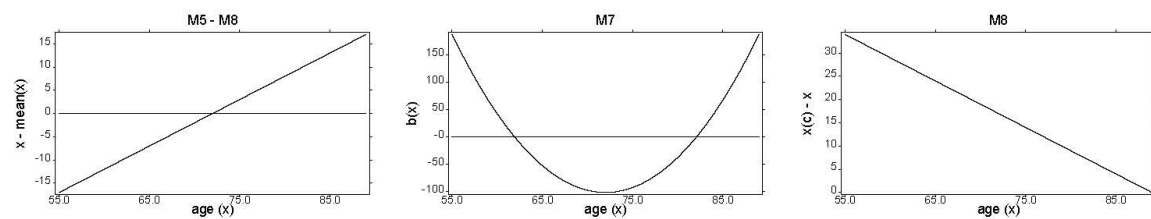
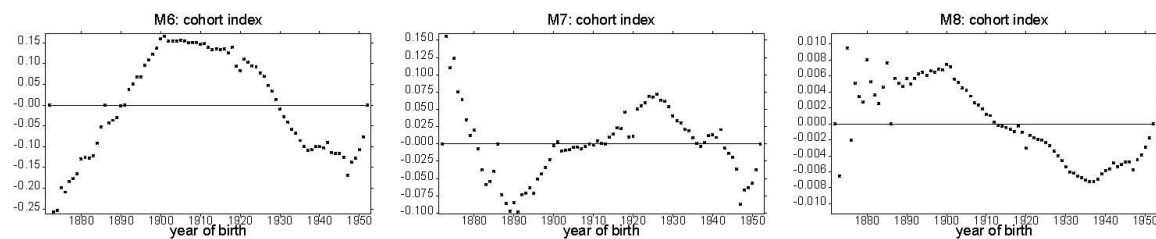


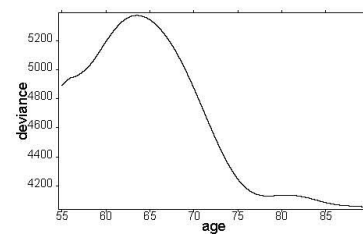
Fig 9. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictors LC, LC2, H1, M.



Models M5-M8: age modulating functions



Models M6-M8: year-of-birth indices



Model M8: deviance profile

Fig 10. England & Wales 1961-2007 male mortality experience, ages 55-89.
Binomial responses, log-odds link, target $q(x,t)$, predictors M5-M8.

SUMMARY

Model	Age effects	Period effects	Cohort effects
M5	x	✓	x
M6	?	✓	✓
M7	?	✓	✓
M8	?	✓	✓
LC	✓	✓	✓
LC2	✓	x	x
H1	✓	✓	✓
M	✓	✓	✓

MODEL DYNAMICS

- Characteristic features of random walk projections.
- Dominant primary period component exhibits a downward trend.
- Primary period component is linear for H1, M and M6 models, but shows curvature for other models.

MODEL DYNAMICS (cont...)

- For M5-M8, the forecast trend in secondary period component varies in direction across models. Also form of $\beta_x^{(1)}$ means that contribution switches direction at mid point of age range.
- For M7, form of $\beta_x^{(2)}$ means that tertiary period components changes direction twice.

REFLECTIONS

- APC models (M and H1) provide good fit; lead to stable estimates and forecasts: use 1 time factor; converge slowly (for M- without adjustment).
- CBD models (M6, M7, M8) provide good fit although cohort effect is less prominent; lead to stable estimates and forecasts; converge quickly; use multiple time factors; lack of α_x term means age range needs to be restricted.

FORECASTING PERFORMANCE

- Use England and Wales male experience for 1961-1982, ages 55-89 to fit models and then calculate life expectancies (and annuity values) for age in 1982 by cohort method where $x=65, 66, \dots, 80$.
- Compare values for same indices calculated using raw mortality rates for 1983-2007, by depicting (predicted-actual) values against age.

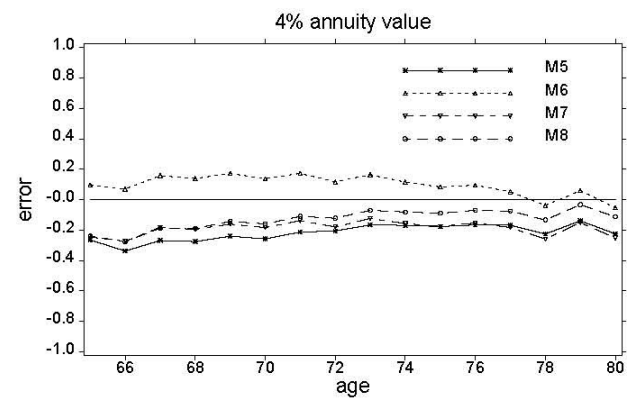
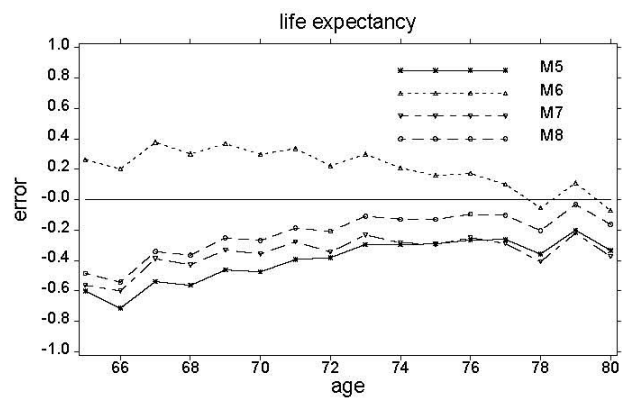
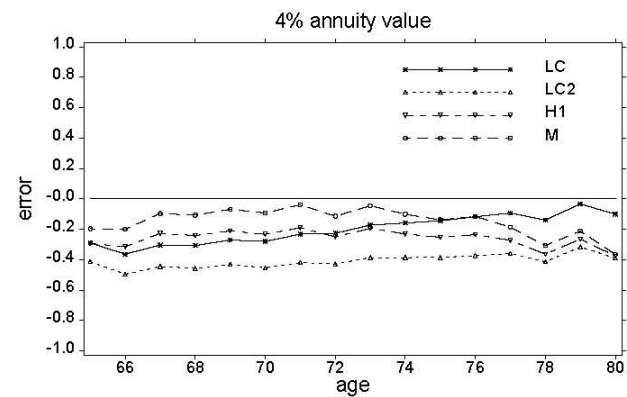
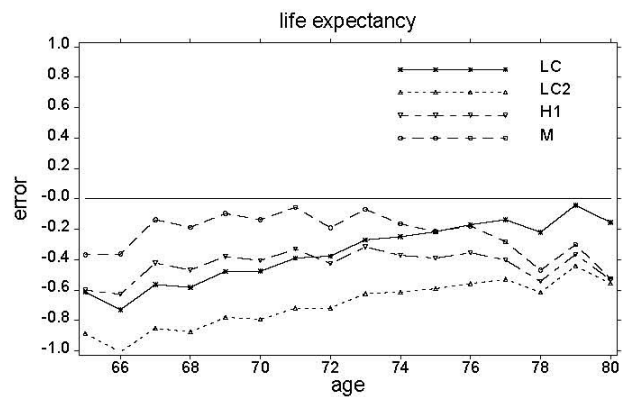


Fig 13a. Retrospective error in 1982 predicted life expectancies (LH panels) and annuity values (RH panels): ages 65-80.

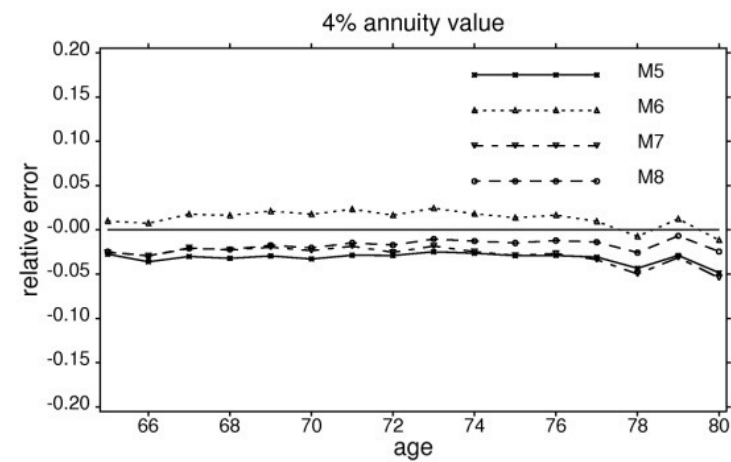
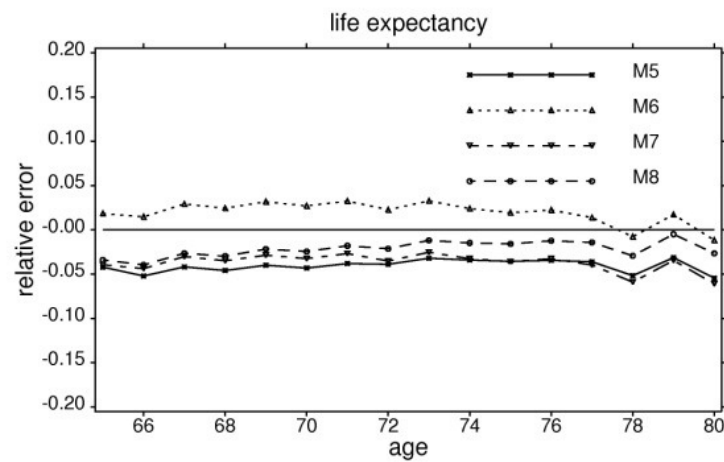
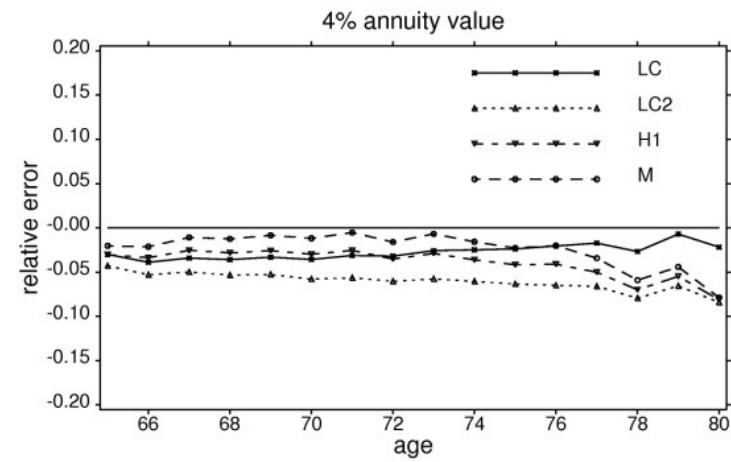
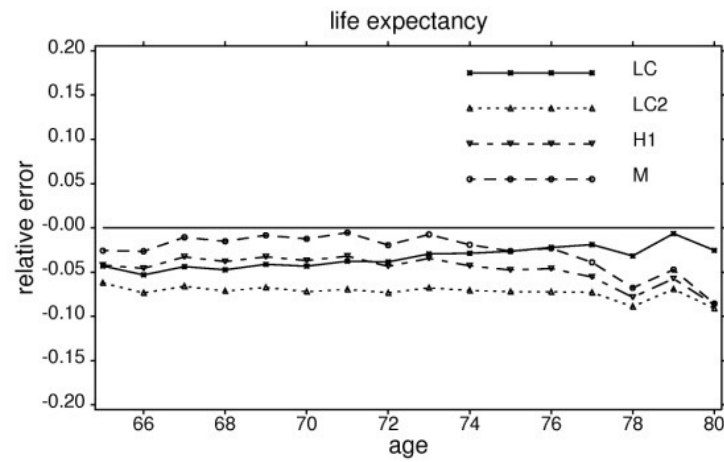


Fig 13a. Retrospective relative error in 1982 predicted life expectancies (LH panels) and annuity values (RH panels): ages 65-80.

Repeat for log mortality rates for the domain bounded by ages 60-89, period 1983-2007 and years of birth 1894-1923: plot errors against age, period and year of birth.

Worst performing models are LC2 and M5.

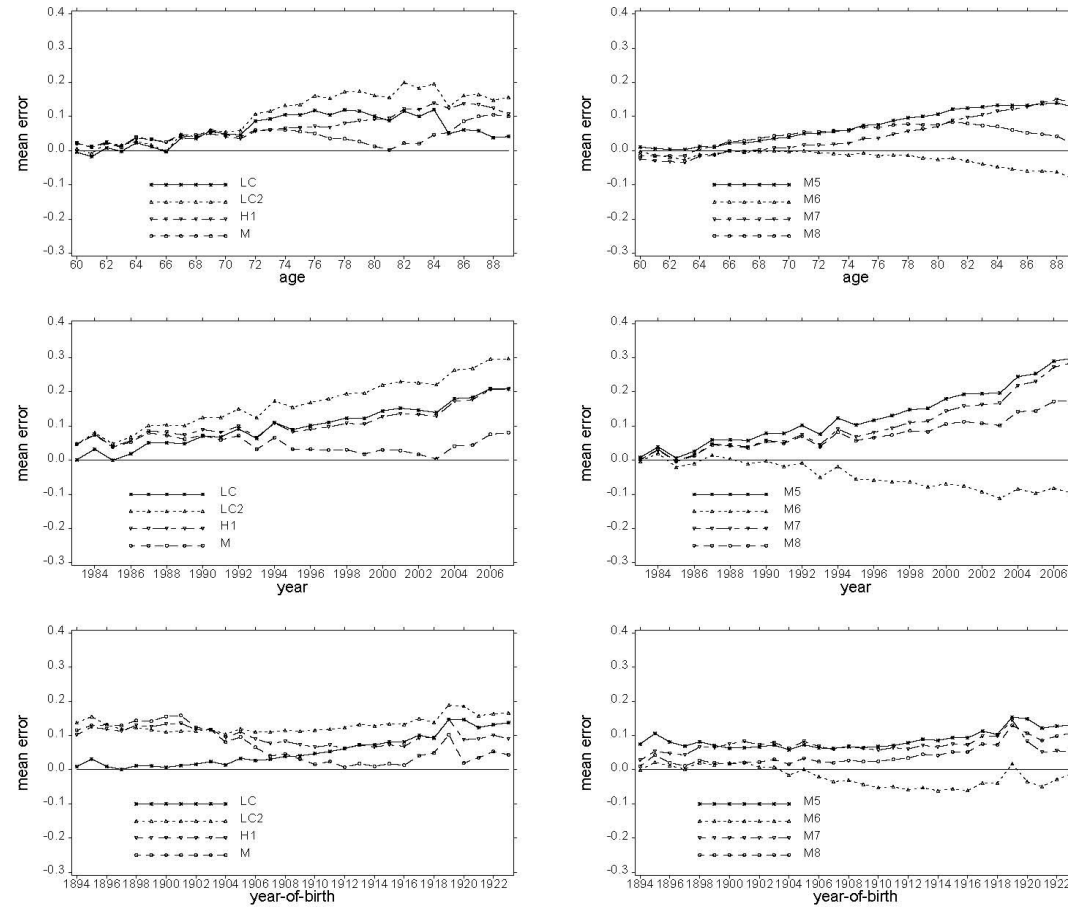


Fig 13b. Retrospective error in 1982 predicted log death rates, averaged over ages, years, cohorts respective (1st to 3rd row of panels): based on the region bounded by ages 60-89, years 1983-2007, cohorts 1894-1923.

WHAT IS A GOOD MODEL?

- Consistency with historical data.
- Parameter estimates and model forecasts should be robust relative to the period of date and range of ages used.
- Forecast levels of uncertainty and central trajectories should be plausible and consistent with historical trends and variability in mortality.
- Model should be straight forward to implement using analytical methods or fast numerical algorithms.
- Model should be relatively parsimonious.

WHAT IS A GOOD MODEL?

- Model should generate sample paths so that prediction intervals can be calculated.
- Model structure should allow incorporation of parameter uncertainty in simulations.
- Where appropriate, model should incorporate a stochastic cohort effect.
- Model should have non-trivial correlation structure.
- Model should be applicable to the full age range.

OTHER POINTS TO NOTE

- Trade-off between goodness of fit and forecasting accuracy.
- Time series methods and their application to long forecasting periods.
- Appropriateness of data sources for particular applications e.g. hedging: adverse selection and “basis risk”.
- Model error – essential to investigate more than one modelling framework. Need to understand the limitations and assumptions of each potential model.
- Sources of uncertainty – process, parameter, model, judgement.
Not all sources of uncertainty can be quantified.

NEW DEVELOPMENTS

- Modelling of mortality improvements (with A. Renshaw)
- Allowing for jumps, regime switches
- Joint modelling of populations (With A. Villegas, P. Hatzoupoulos, A. Debon)
 - different countries, different subgroups
 - coherence and cointegration
- Allowing for additional covariables
 - macroeconomic factors
- Improved methods of bootstrapping (with V. D'Amato)
 - including allowing for dependence.